GETTING AND KEEPING CHILDREN ENGAGED WITH A CONSTRUCTIONIST DESIGN TOOL FOR CRAFT AND MATH

A Dissertation
Presented to
The Academic Faculty

by

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In Partial Fulfillment
of the Requirements for the Degree
Doctor of Philosophy in the
College of Computing

Georgia Institute of Technology
May 2007

GETTING AND KEEPING CHILDREN ENGAGED WITH A CONSTRUCTIONIST DESIGN TOOL FOR CRAFT AND MATH

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To Lambo

ACKNOWLEDGEMENTS

First and foremost, I wish to thank my family for all of their support throughout the process of my education. Without their love, guidance, and moral support, I could not have gotten this far. I especially thank Lambo and Maura for their patience and understanding.

I wish to thank my friends at Georgia Tech – especially all the potluck friends who kept me entertained through board games and fed with yummy exotic foods, and the happy hour crew whose weekly gatherings gave me a chance to relax.

I wish to thank Elaine Huang, Joe Tullio, and Duke Hutchings for being good neighbors *and* colleagues.

The students and faculty in the Learning Sciences and Technology group at Georgia Tech provided many useful conversations, a great deal of useful feedback, and lots of help along the way. I want to thank Tammy for being a positive voice and for always offering to help. Thanks, and several hours of time, are certainly due to Allison Elliott Tew, Brian Dorn, and Jochen Rick for helping me with last minute details.

I would like to thank my colleagues at the University of Minnesota, Morris, for their general support in finishing this document, and Sue Gilbert and Nic McPhee for their more specific help in "gathering" my milestones without adding undue pressure. I am grateful to Tisha Turk who provided lots of feedback on the structure of this dissertation at a critical time and helped keep me motivated. Talking with Tisha about my work clarified a lot of things for me and helped me improve my writing.

Many thanks are due to Megan Chinburg, Marc Callahan, Jeffrey Rosellen, and especially Jochen “Je77” Rick for help with the DigiQuilt software along the way. I would also like to thank Kristin Vadas for her careful and thoughtful transcription of numerous hours of video data.

I want to thank Idris Hsi for so much I don’t know where to start – everyone needs their own “Idris” when they are working on their doctorate. I would like to thank Andrea Forte for being a wonderful, helpful, knowledgeable, and supportive friend, in life and in work, at all the right times.

I would like to thank the members of my committee for their help in forming and shaping this dissertation. Mark Guzdial and Amy Bruckman provided constructive feedback and guidance from the early days of this project through the defense, and Ron Ferguson and Mike Eisenberg added useful, creative ideas in the later formative stages. I hope these conversations can continue in the future.

Finally, I would like to thank my advisor, Janet Kolodner, for taking chances and exploring this thread of research with me. Her countless hours meeting with me over ideas, papers, and hot chocolate helped me grow as a researcher and as a person. It seems

that we both learn by talking things through, and we have certainly done a lot of that in the past few years.

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SUMMARY

Manipulatives of various kinds are used in elementary schools as part of the mathematics curriculum. They are recognized for their affordances for helping children understand abstract ideas by connecting them to concrete objects, but not all research about the use of manipulatives has been positive. It is sometimes difficult for children to make connections between what they are doing and the ideas the manipulatives embody.

Constructionist research suggests that taking a design approach to learning that involves the learner in constructing not only ideas but also public artifacts facilitates learning particularly well (Papert, 1991). Further, it suggests that one should design a learning environment that will allow learners to leverage personal and epistemological connections rather than scripting everything that should happen (Resnick et al., 1996). Additionally, previous research suggests that integrating math with craft and design helps learners engage with math in a personally meaningful manner (e.g., Eisenberg & Eisenberg 1997, 1998; Shaffer, 1997; Elliott & Bruckman, 2002).

Manipulatives such as pattern tiles and quilt builder tiles are used for design in the classroom, but there is often little to no support for the analysis of designed patterns or other kinds of learning. On the other hand, computerized versions of manipulatives that provide feedback about fractions (or other math concepts) do not offer affordances for design. My goal has been to integrate the best of what computational manipulatives can offer with a design approach to help learners engage with math in a meaningful way.

In this dissertation, I describe the use of a manipulative that combines affordances for design and also links and maintains connections between representations. This gave

learners opportunities to see and make connections between symbolic and concrete representations while engaged in designing personally meaningful artifacts. I describe methods that made this constructionist educational experience accessible to a wide range of learners, including aspects of the socio-technical system that seemed to play greater or lesser roles at various times throughout the study. I emphasize roles of the DigiQuilt manipulative and highlight how this software builds on previous work, yet represents a new kind of manipulative – one that simultaneously supports design and connecting targeted math concepts with concrete artifacts.

CHAPTER 1

INTRODUCTION

In Seymour Papert's *Mindstorms* (1980), we read about his childhood fascination with gears. He wanted to know everything about gears, and he spent much of his time exploring and learning with them. His passion for gears motivated him to continually want to find out more. How can we encourage this kind of extended engagement for children? How can we spark an initial interest and support long-term engagement with a learning activity?

Many people express themselves through craft, art, and design[1] activities. It is something people are willing or even excited to spend their free time doing, and the artifacts they create are often cherished, not only in the moment, but also for many years to come as an heirloom. Quilting is one such medium. For me, part of the beauty of quilts is the math. I want to help people appreciate quilts on another level by helping them notice the mathematical beauty of quilts. I want to help children mathematize[2] their worlds.

Constructionism, an approach to education based on the constructivist theory that people learn by building their own knowledge, suggests that people learn "most felicitously" when they are constructing artifacts for an audience (Papert, 1991). Further, constructionist research suggests that designers of learning environments should focus on providing affordances for learners to leverage personal and epistemological connections (Resnick et al., 1996). Previous research (e.g., Kafai & Harel, 1991; Kolodner et al.,

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[1] In this dissertation, I use art, craft, and design interchangeably because "art" in the elementary school seems to be a combination of these.

[2] By mathematize, I mean that I want to help children see math in the world around them – to be able to see the mathematical nature of everyday things.

2003, 1998; Shaffer, 1997) not only suggests that a design approach will be *engaging* for learners, but also suggests that it provides opportunities for learners to leverage personal and epistemological connections. How can this concept be applied to math in school? What kinds of support are needed in order to help learners leverage epistemological connections while at the same time allowing them to design personally meaningful artifacts?

One goal of my research is to look at ways we can get and keep children engaged with a constructionist design tool for craft and math – how we can make a constructionist experience accessible to the variety of learners in a classroom setting, while still supporting students who want to delve more deeply and would sustain the learning experience by their own choice. How can we support learning for everyone, and passion for some? How can we help children notice math in their worlds?

A microworld approach to mathematics lets learners construct and interact with powerful mathematical ideas within, a virtual "Mathland" (Papert, 1980) on the computer. Eisenberg (2003b) talks about bringing math into the world by having children create *physical* mathematical artifacts, so that rather than interacting with "Mathland" only on the screen, we might allow children to experience math in their everyday lives. This idea of bringing mathematical artifacts off the computer and into the world is inspiring to me, but I would like to also help children see math that already exists in the world around them as well.

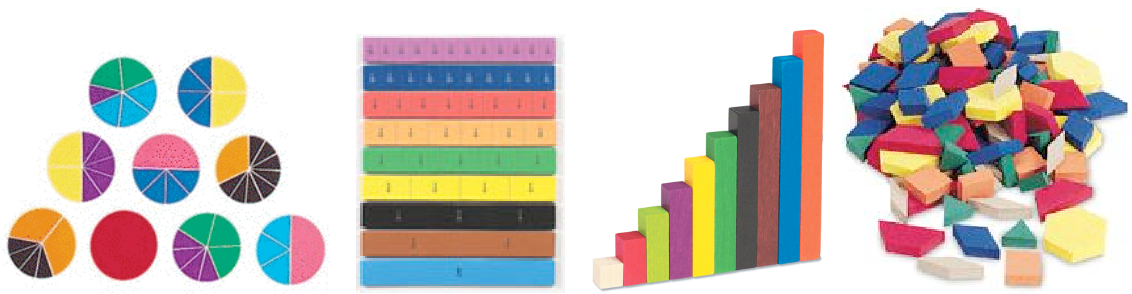
In this dissertation, I will describe my first steps towards helping children mathematize their worlds. For this research, I created a mathematic design manipulative environment for children to create patchwork quilt blocks and connect their designs to fractions and symmetry concepts in the process. The software system, called DigiQuilt, that is part of this environment represents my efforts to combine the affordances of two different kinds of manipulatives – ones that support design, and ones that support connecting concrete objects with the abstract mathematical ideas they embody. Children using the system were able to create personally meaningful designs and connect their designs to the abstract mathematical ideas of fractions and symmetry. They engaged with the creative aspects of the system to create quilt blocks depicting things from their lives, and they shared these designs with other people including teachers, peers, and their families. They persisted through difficulties to achieve goals they set for themselves, as well as to solve challenges they were given. They found support in the socio-technical system to connect their designs to fractions and symmetry, and they talked about those connections, helping each other out along the way.

Manipulatives

The American Heritage Dictionary of the English Language defines manipulative as follows:

> **ma·nip·u·la·tive**
>
> *n.* Any of various objects designed to be moved or arranged by hand as a means of developing motor skills or understanding abstractions, especially in mathematics.

Manipulatives of all kinds are used in elementary schools as part of the mathematics curriculum. They are recognized for their affordances for helping children understand abstract ideas by connecting them to concrete objects. Some manipulatives focus on one particular mathematical concept (e.g., fraction pies or bars (see Figures 1a and 1b)), and some are used for multiple concepts (e.g., Cuisenaire rods (see Figure 1c) which are used for fractions and for integer math, or pattern tiles (see Figure 1d) which are used mostly for making geometric designs, but which also have activities that help teachers use them for fractions lessons). While manipulatives are widely used in elementary school math, research results regarding how successfully they are supporting learning have varied (Thompson, 1992; Sowell, 1989; Moyer, 2001).



(a) (b) (c) (d)

Figure 1. (a) Fraction pies, (b) fraction bars, (c) Cuisenaire rods, and (d) pattern tiles.

Computational Manipulatives

In recent years, there has been an effort to make manipulatives available in electronic format (e.g., Educational Java Programs (Bulaevsky, 1997), National Library of Virtual Manipulatives (http://nlvm.usu.edu/en/nav/vlibrary.html), Visual Fractions (Rand, 1998), ExploreLearning (http://www.explorelearning.com/)). These "virtual manipulatives" are recognized for some affordances they have above and beyond physical manipulatives (Kaput, 1992; Moyer, 2001; Sarama, 2004). Whether it is simply having unlimited pieces that can't be "lost" or linking and maintaining relationships between various representations, most virtual manipulatives take advantage of at least some of the power of the computer to support learning. For example, the National Library of Virtual Manipulatives (NLVM) has some amazing utilities for working with pattern blocks that allow children to explore a wide variety of transformations (translation, reflection, and rotation) designs. Learners can create designs by selecting pieces to place on their designs. Depending on which tool is in use, there are different capabilities available to support learners as they explore different geometrical transformations. The NLVM also has a variety of applets for exploring fractions. In the 3-5 and 6-8 grade sections, there are applets dedicated to exploring equivalent fractions, comparing fractions, multiplying fractions, and building fractions using bars or pieces of a circular representation. These programs allow children to manipulate a visual representation (bars of varying lengths, or pieces whose areas correspond to the amount of the "whole" (i.e., a spatial ratio)) and view the symbolic representation that matches it. Other computer-based fractions rods or similar manipulatives offer feedback about the fractions learners create when they combine rods.

I believe such computer-based manipulatives can be greatly improved. One idea is to offer multiple kinds of feedback for the more versatile manipulatives (the ones that have already been marketed for several uses). Another is to provide some kind of context for using the manipulative that puts learners in situations that are likely to lead to learning. It makes sense to utilize the power of the computer in other ways while learners are engaged with computational manipulatives, for example, to aid collaboration, to guide learners to learning opportunities, and to help learners focus on important aspects of their interaction with the manipulatives. Finally, it seems to me that combining the affordances of manipulatives for design and manipulatives that emphasize connections between concrete objects and the abstract mathematic ideas they embody would benefit learners.

Manipulatives for Design

One context for the use of manipulatives that I believe has affordances to promote both extensive learning and engagement is design. Design has been recognized as a motivating vehicle for learning (Harel, 1991). Constructionist research suggests that taking a design approach to learning that involves the learner in constructing not only ideas but also public artifacts facilitates learning particularly well (Papert, 1991; Resnick et al., 1996).

Additionally, previous research suggests integrating math with craft and design to help learners engage with math in a personally meaningful manner (e.g., Eisenberg & Eisenberg 1997, 1998; Shaffer, 1997; Elliott & Bruckman, 2002). There are several types of manipulatives for creating and learning about patterns and geometry (e.g., pattern tiles, quilt builder tiles, and “Fractiles”) available for learners. These physical manipulatives lend themselves well to design activities, but the designs that are created are difficult to share (which constructionist researchers suggest is important). Additionally, difficulties arranging the pieces and running out of pieces can frustrate learners.

Computational Manipulatives for Design

Computational manipulatives for design alleviate some of the difficulties associated with their physical counterparts. However, previous versions of computational manipulatives for design have not also provided explicit support for other kinds of learning while the learner is still engaging with design. For example, computerized versions of pattern blocks allow learners to make interesting patterns, but many do not explicitly support the design process itself, and there is little to no support for the analysis of designed patterns or other kinds of learning. On the other hand, computerized versions of manipulatives that provide feedback about fractions (or other math concepts) do not offer affordances for design. In other words, there were no computational manipulatives that offered explicit support for both design and learning from design. My goal in designing DigiQuilt has been to integrate the best of what computational manipulatives can offer with a design approach to help learners engage with math in a meaningful way. I think that a manipulative that combines affordances for design and also links and maintains connections between representations will give learners opportunities to see and make connections between symbolic and concrete representations while engaged in designing personally meaningful artifacts.

DigiQuilt

DigiQuilt (Figure 2) is the computer-based construction kit I designed and built for exploring computational manipulatives. It supports children's learning about math and art as they design patchwork quilt blocks (Lamberty & Kolodner, 2002). Learners create quilt blocks by selecting pieces (colored shapes) from a palette and placing them in one of the work areas. The pieces snap into place in patches (grid-sized squares, which can be made of multiple pieces). The software offers learners 4-, 9-, or 16-patch base blocks (frameworks for constructing quilt blocks) with a variety of grids that can be superimposed on them to support looking at the quilt in multiple ways. There are facilities for saving and reloading quilt blocks and buttons for clearing the block work area and stepping forward or backward through a design's history. The ability to save and reload designs allows learners to easily share their designs or start from previous designs. Navigating through a design's history allows learners to see how it was made or undo "mistakes" made while creating a design. Learners have access to a palette of shapes (pieces) with buttons to change their colors, and facilities for rotating pieces or patches and copying or swapping patch-level patterns so that they can be easily repeated or changed. In DigiQuilt, learners move pieces and patches to create their quilt block designs; much like a physical manipulative. In addition, learners can see what fractional area of their quilt block design is covered with each color in the palette. This feedback is provided as a reduced fraction located on the button of the corresponding color. Designs can be printed or saved easily, and they can provide context for spontaneous and teacher-led discussions of the targeted concepts.

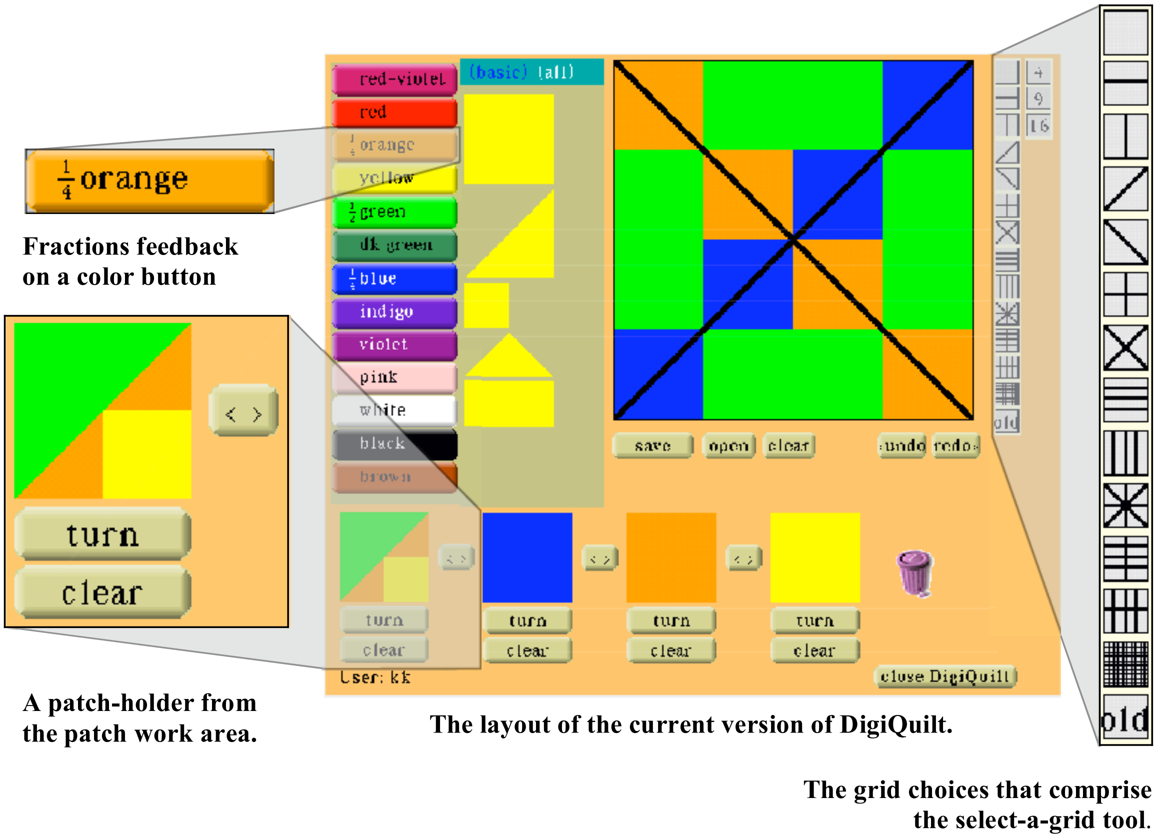


Figure 2. A screenshot of DigiQuilt.

Learners using DigiQuilt in the classroom create quilt blocks that solve challenges that ask them to focus on fractions and symmetry, e.g., "Make a quilt block that is 1/2 one color and 2/4 some other color." The challenges are developed with specific learning goals in mind. In this example, the goal is to introduce equivalent fractions; a concept that is particularly difficult for many children to understand (Arnon et al., 2001). The feedback the software provides about fractions does not limit the students' design activities, but provides a way for students to monitor progress toward their personal goals or the goals set forth in challenges provided by the teacher. By providing feedback coupled with challenges that are likely to lead to outcomes that differ from what learners generally expect, learners will have many opportunities to experience "expectation failure" and adjust their knowledge structures accordingly.

DigiQuilt has similar affordances to other computer-based manipulatives and microworlds in that it continuously updates the relationships between representations, allowing the learner to focus on the effect their actions have on the math of the design (Clements & Battista, 2000), but it goes beyond helping students build bridges between the concrete and abstract by integrating opportunities for creating personally meaningful designs. Notice the parallel – constructionist research suggests providing opportunities for learners to leverage their epistemological and personal connections. That is the goal with the DigiQuilt socio-technical system – the design aspects allow learners to leverage their personal connections by making things they care about or are familiar with from the world, and they can leverage epistemological connections with the support of the challenges and tools in the software that are available for linking the concrete artifacts to the abstract ideas they embody.

A New Kind of Manipulative?

How can we leverage computational power to enhance the learners' experience with math manipulatives? How can different lenses and modes of interaction affect the learners' experience with design manipulatives? What roles can different aspects of the socio-technical system play in supporting learning through the use of design manipulatives?

Hypothesis

This research explores the effects of leveraging computational and environmental support for learning through design manipulatives. I aim to show the power of building a network of technical and social supports around a manipulative so that learners can construct, understand, analyze, explore, and share designs they create. This work aims to answer the question: how can we leverage the power of computational manipulatives to help learners see math in the world, bring math into the world, and begin to mathematize situations in the world around them? Through my work on answering this question, I believe that I have come a long way toward defining a set of design principles for computational manipulatives that will lead learners to rich learning experiences.

My hypothesis is that integrating affordances from design manipulatives and manipulatives that provide strong links between concrete objects and the abstract mathematical ideas that they embody will result in a manipulative environment that allows children to simultaneously engage with design activities and make connections between their artifacts and targeted math content.

Notice the parallel between the hypothesis and suggestions of constructionist research. Here, the design aspects of the socio-technical system are providing affordances for personal connections, and the support in the socio-technical system for noticing connections between the concrete artifacts and abstract targeted math content is providing affordances for epistemological connections.

Goals and Predictions

My hypothesis relates to several overarching goals and leads to two sets of predictions related to personal and epistemological connections. The overarching goals of this research are to help children talk about math and see math in the world around them. In order to help children talk about math, I believe it is important to help them see and understand things in a mathematical way. To help children learn to mathematize things in their world, I believe it is important to help them build bridges between “school math" and objects in their lives. We need to give them opportunities to talk about math, and we need to make these experiences meaningful and interesting – and make sure they continue to have these experiences that help them see and build connections between math in school and math in the world. Through repeated opportunities to experience value in talking math, I hope children will develop the disposition to see the world in mathematical ways and to express themselves mathematically.

My predictions stem from the idea that combining “school math” and art through quilting with a computational manipulative that supports design will allow us to leverage both the affordances of learning through design (personal connections) and the affordances of using manipulatives to learn math (epistemological connections). Research on learning through design tells us that a design approach engages learners. The math literature suggests that manipulatives are most effective when they help learners connect the concrete with the abstract. To show that combining math and art through designing quilts allows us to leverage the affordances of design and the affordances of manipulatives for helping children connect the concrete and abstract, I will need to show that children were able to leverage both personal and epistemological connections. This leads to two sets of predictions.

Predictions from the “design” part of the hypothesis (personal connections)

The children will be engaged with the design manipulative. They will be excited about sharing their designs with family and friends. They will persist through difficult moments. They will enjoy using the software to express themselves. Children will:

1. Be excited to share their designs,
2. Persist through difficult moments, and
3. Use the software to express themselves.

Predictions from the “bridging” part of the hypothesis (epistemological connections)

The children will connect the concrete quilt block designs they create with the abstract “school math.” By helping them view these artifacts in a mathematical way or interact with these artifacts using tools that help them reflect on the artifacts in a mathematical way, we can help children notice and talk about the math in their designs. Children will:

1. Connect their quilt block designs to fractions and symmetry, and
2. Find support in the socio-technical system for noticing math and making these connections

Context

To inform my hypothesis, I have conducted three field studies over the course of three years at three different schools near Atlanta, GA. The first two field tests served mainly to inform the design of DigiQuilt. For each trial, I had children work with the software or a paper version of the software to design patchwork quilt blocks that addressed challenges I gave them (e.g., "Design a quilt block that is 1/2 red and 1/2 blue.") Working with third through fifth graders (approximately 8-12 years of age) helped me understand more about what learners of this age find difficult with respect to understanding fractions and symmetry. I developed a variety of strategies to help them connect these abstract ideas to the concrete world around them (e.g., structuring activities in different ways and adding software tools and supports for creating designs and learning content). After each field trial, I made changes to the software and activities to address these findings. I found that students were eager to share the designs they created and that sharing their designs in different ways often motivated them to work with the software. They often persisted through difficulties they had when trying to make their designs look a certain way. They created designs that represented things from their lives, and told stories about their quilt designs. I found that the software helped children create quilts that solved the challenges. Additionally, the social context and tools in the software supported students' efforts to describe math in their quilts, and seemed to help them understand some of the math in the challenges I gave them. The third field trial (consisting of four semi-simultaneous (staggered-start) classroom studies) initially focused on understanding how to get and keep children engaged with a computational manipulative for design, but changed slightly as it progressed and began to tell me so much about *how* children were engaging within the socio-technical system, and what kinds of supports they seemed to need to bridge between the artifacts they were creating and the mathematical aspects of those artifacts. Though I did not formally measure learning, the participants' successes with the software and social supports in the system illustrated that children were engaging with math in ways that lead to learning. In this dissertation, I present the procedures used in the third field trial, focusing on the fourth classroom study I conducted as part of this research.

Contributions

The work completed in the process of writing this dissertation will contribute to the learning sciences and technology field in two ways. First, it offers suggestions for (and an example of) a new kind of computational manipulative, including a description of its use in the field. Second, it offers an example of a constructionist experience within a current classroom setting that was accessible to many learners, and allowed some learners to delve more deeply in their free time – perhaps discovering a new passion or hobby.

To tell the story of this research and its results, I will first describe the background for this work. Next, I will introduce details about my computational manipulative, DigiQuilt, along with its design history. After that, I will share the context and story of my dissertation study (including my research methods and results of the study). Then, I describe how I dealt with the data and present an analysis of the data I collected in the field. Finally, I situate this manipulative, describing ways it is similar to and differs from two previous schools of thought on manipulatives, and make suggestions for the design of computational manipulatives.

CHAPTER 2

BACKGROUND

Before developing DigiQuilt, I had the idea that quilting would be a good medium for exploring mathematical concepts. Additionally, I imagined that combining art and math would allow more children to enjoy learning with the software (since children with either preference could find something of interest). Combining information from literature on a variety of educational approaches, learning theories, and previous work, (constructionism, learning through design, and math learning), I developed software for children to design patchwork quilts and learn about fractions and symmetry in the process. In this chapter, I will describe some of theories and previous work that guided my research program and influenced the design of the DigiQuilt software and surrounding socio-technical system.

Constructionism

The educational approach that has had, perhaps, the most impact on the design of DigiQuilt software is constructionism. Constructionist research builds on the constructivist theory that learners build knowledge structures for themselves through an active process of assimilation and accommodation. It further suggests that students learn "most felicitously" when they are consciously constructing artifacts for an audience (Papert, 1991). From a meta-cognitive perspective, when learners consider how someone else will interpret or understand whatever they made, that consideration influences their own thoughts about the embodied concepts. Including an audience in the cycle between the internal knowledge structures and external artifacts causes learners to consider how the audience understands or perceives the constructions.

The importance of shared artifacts in this educational approach allows an inclusive epistemological view. Whether someone creates a rocket ship by starting with a plan and building each part in a certain order, or painting streaks of red and declaring that it looks like fire spewing from the back end of the ship, each approach allows the learner to experience the rocket. While many theories place highest value on abstract ways of knowing, constructionism embraces the *bricoleur* who experiences successful learning through a particular closeness to objects – interacting with the very objects that embody abstract ideas (Turkle & Papert, 1992).

Constructionism supports the idea that there are multiple ways of knowing and learning. Turkle and Papert suggest that supporting these bricoleur designers is one step toward a "revaluation" of the concrete (1992). If anything, DigiQuilt is more welcoming to bricoleurs than to abstract thinkers since direct manipulation of objects is required (though both planful and emergent designs can be made). I hope to support learners as they build bridges between the abstract and concrete by using a design approach to learning where the software manages relationships between representations.

In *Mindstorms* (Papert, 1980), we read about Papert's childhood fascination with gears; it was something he wanted to understand completely, and he delved into this learning with such a passion that he came to understand them quite deeply. Most traditional educational settings do not easily accommodate a purely constructionist approach because while it does not suggest that instruction itself is a problem, constructionism does not tend to approach instruction according to a set curriculum. Some critics are concerned about introducing such an approach in schools because they fear that many children would fail to learn all the things they need to know. Papert suggests that all children have something that could, for them, give rise to such an interest in learning. Introducing students to potential "gears" in the classroom so that they may delve deeply into a "gears-like" learning experience of their own, then, is a worthwhile goal. When critics point out that this model for learning does not fit with the current educational systems, Papert suggests that is actually a problem of the school system – constraining children's activities to gradable, structured activities with their age peers (Papert, 1998).

In response to both proponents and critics of constructionism, one aim of this research is to show how we might help some children find their "gears" while offering an accessible learning experience for all the students in the classroom. To do this, I have created a design environment with opportunities for deep exploration and creative engagement, but also tools that highlight certain aspects of the quilt blocks that children design in the system. In that way, I envision the software as part of a socio-technical system that includes some activities that steer students toward predetermined learning goals that are best supported by the quilting medium and the software. However, I anticipated that some children would be motivated to go above and beyond the challenges I provided. In that case, there are tools to make beautiful and complex quilts while still getting feedback on the targeted math content of fractions and symmetry.

Learning Through Design and Expressive Mathematics

"Design motivates learning. Before learning and productive thinking can occur, people must be motivated. Motivation to learn and think in general, and in mathematics in particular, depends on recognizing that something is important, that it is relevant to oneself" (Harel, 1991). Research on learning through design and expressive mathematics emphasizes benefits of using various aspects of a design-centered environment for learning. Research in this area describes ways in which students can learn by working together around design artifacts, and what kinds of support they may need along the way.

Children and Design

In the Instructional Software Design Project (ISDP), 3rd, 4th, and 5th graders participated in different aspects of the design process, acting as learners, designers, and consultants (Kafai & Harel, 1991). The 4th grade designers worked over a long period of time on their software, which was aimed at teaching younger students about fractions. The audience was relatively non-threatening and provided motivation for thinking about fractions in terms of what was hard to understand, allowing students to admit difficulties they had and use them to their advantage rather than feeling defensive about them. I like this idea, and though I placed less emphasis on this, I initially helped children adopt this meta-cognitive approach when they were using DigiQuilt.

Supporting children in learning through design activities requires some special considerations. (Siraj-Blatchford & MacLeod-Brudenell, 1999) tells us that there are two issues that are often confused: supporting the child in the design endeavor itself and concern for the child to record and communicate their design ideas to others. The authors emphasize the importance of supporting the child as they explore and model, sometimes using actual materials rather than drawing out plans and acting on them (at least at first when ideas can be easily lost in imperfect recording or communicating). In other words, we should encourage children to make and carry out plans, but not at the expense of losing their ideas in the design process. I take this notion to heart. Luckily, planning and creating designs in DigiQuilt can basically be one and the same since the direct manipulation of pieces and patches is supported. In this way, ideas are unlikely to be lost in the shuffle. On the contrary, some new ideas might even be created in the shuffle.

Learning By Design

In Learning By Design™ (LBD) (Kolodner et al., 2003, 1998), students collaborate to plan, design, build, and test models, and analyze their results in order to solve challenges.

LBD is based on case-based reasoning – the idea that learning occurs through a process of applying lessons learned from previous experiences (or cases) in novel situations, looking at the outcomes, and indexing and re-indexing experiences in memory based on the outcomes. Because of its roots in case-based reasoning (Kolodner, 1993), LBD focuses on having students read about what other people have done, think about how their results might be relevant to others, and strive to share information about what they have done so that others may also learn from their experiences. Sharing and comparing cases to each other in productive ways is a large part of the learning process in LBD. In LBD, students work on relatively complex design challenges and present their design ideas to others before and after testing them, allowing them to learn from each others' designs as well as their own.

I agree with the idea that learning through deliberating about a variety of cases is important, and that doing that deliberation collaboratively provides even more opportunity for understanding the cases since learners need to interpret their own experiences to figure out what is important to tell others in addition to trying to understand the experiences of others. When I work with children in the classroom using DigiQuilt, I encourage them to discuss their quilt designs with each other and think about how those designs solve the same challenges in different ways. In DigiQuilt, therefore, I made it easy for learners to see artifacts created by other users and even follow the steps that were used in the construction of those designs. Through discussing, describing, exploring, and sharing their designs, learners begin to consider the math in their quilts. Since the design histories are saved along with the artifacts, both the artifacts and the process of making them become part of the shared learning experience – a part of the set of cases that are available for future reference. Getting their ideas into the world where they can be challenged and explored is a key component to tackling misconceptions.

Combining ideas from several areas

Research on constructionism (e.g., Papert, 1991, 1980; Resnick et al., 1996) and learning through design (e.g., Kolodner et al., 1998, 2003; Shaffer, 1997) suggested that a design environment would be engaging for learners, and that it would provide the kinds of experiences and connections that make for a good learning environment. Resnick, Bruckman, and Martin (1996) tell us that creating design-based learning environments is not a task where we can plan every detail of the learning experience. Instead, they suggest that the challenge is to, "create frameworks from which strong connections – and rich learning experiences – are likely to emerge." To do this, they offer an important guiding principle: connect kits and activities to users' interests, passions, and experiences, *and* to important domains of knowledge. In other words, allow the learners to *leverage* their experiences through the personal connections, and make sure the connections to the knowledge are salient enough to allow them to learn about the domain *through* their designing and creating.

Expressive Mathematics

Several research projects have sought to bring art and math together in various ways with different goals in mind (e.g., Shaffer, 1997, Eisenberg, 1997, Elliott, 2005). Shaffer (1997) describes a design studio environment called Escher's World where investigations, explorations, and peer review in a relaxed setting encouraged learners to explore mathematics expressively. High school students who participated in Escher's world started to use visual problem solving strategies and reported liking math more after only 12 hours of participation. The nature of the dynamic representations on the computer helped students understand complex ideas and gave them a sense of control over their work. Shaffer suggests that an interesting and important aspect of allowing students to engage with math in an expressive way is that learners with a range of interests can interact with mathematical ideas on their own terms. I like the idea that allowing students to engage with math in an expressive way may help them approach math on their own terms. I try to give students an expressive approach to math, but since my system is aimed at a younger audience, I have decided to keep it more constrained as far as how students can interact with the designs and what they are allowed to design. These constraints are in place to help the students learn the targeted content, and I hope that it still allows them enough freedom to be creative and experiment with math and art ideas through their designs.

Another example of a system that focuses on having learners work with math in an expressive way is HyperGami. HyperGami (and JavaGami, a close relative of HyperGami) users design customized three-dimensional polyhedra on the computer screen (Eisenberg & Eisenberg, 1997). (Eisenberg & Eisenberg, 1998) explores the benefits of using a range of media, the social role the objects can play (souvenirs, expressions of affection, ornaments, gifts), and the effect of having objects present in daily life. They suggest that using craft media for learning math motivates students because students get to participate in artistic, personal expression, and because of the possible longevity of the artifacts, which can often be displayed publicly and shared. This longevity idea particularly struck me since quilts are often considered heirlooms. Though I have not had the children actually sew the quilt blocks they design, the possibility of a lasting, expressive artifact is enticing. Further, (Eisenberg, 2003b) talks about bringing math into the world by having children create physical mathematic artifacts, so that rather than interacting with “Mathland” (Papert, 1980) only on the screen, we might allow children to experience math in their everyday lives.

I like this idea of bringing math into the world, and I decided to do this in two ways. One way, as suggested by Eisenberg (2003b), is by bringing mathematically inspired artifacts into the world. This inspired my idea to allow children to share their quilt designs using stickers or cards that they can trade since collecting things can be a big draw for children. The student participants in my most recent study really enjoyed trading their business cards during free time at school, and they were so popular that one card was even stolen (which was odd since the students could request any designs they wanted even if another student had created it). DigiQuilt designs have even been used as ceiling tiles (Jochen Rick, personal communication, see Figure 3).

Figure 3. DigiQuilt-designed ceiling tiles (Photo courtesy of Jochen Rick).

The other way of bringing math into the world, though, goes beyond this idea. Because the tools in DigiQuilt allow learners to superimpose mathematic structures on their designs to see them in different ways, I think students may develop new "math lenses," ways of looking for and noticing math, they can use to view the world. I think it is important to provide filters or frameworks (I call these "lenses") that highlight certain features of the artifacts being examined. These lenses provide the opportunity to practice looking at something in a particular way, thus suggesting the possibility of looking at something from that perspective. For instance, a tool that could help you understand more about toy tops and how they spin might provide a lens that helps you see how much the top is wobbling with respect to the surface on which it rests, as well as a lens that shows a view from above so that one can examine how much the top is moving around on the surface.

I think that tools that allow learners to see the same artifact through a variety of lenses, used in the context of challenges that ask them to *explore* these different perspectives, will help children learn more about how to mathematize everyday things so that math will be all around them in the world. In particular, lenses for design manipulatives could be used to help learners carefully examine their creations in terms of different targeted math content. The lenses might emphasize the idea that we can tell different mathematical stories about a given situation. For example, that we can tile the floor so that it is half white or (leaving the tiles in the same arrangement) talk about the floor as having 50% of its area covered with white tiles might be made clearer if we can look at the tiled floor and see that even when the story changes, the amount of white tile stays the same. Noticing this lack of change could be an important step towards understanding the sameness of one half and 50%.

Math learning

In this section, I will describe relevant background from the math learning literature. I will begin by introducing some general themes from the math learning literature, and then go on to talk about children learning fractions and symmetry, and the use of manipulatives for math learning.

The math literature says that understanding mathematics “involves recognizing relationships between pieces of information” (Hiebert & Carpenter, 1992). Children need to see that mathematics is about making sense of their world, and an important part of sense making is connecting everyday informal experiences to formal mathematical language, notations, and methods (Fuson, 2004). Practical consideration along with psychological and educational research suggest that in order to make that happen, an investigative approach is best for, "promoting all aspects of mathematical proficiency: conceptual understanding, computational fluency, strategic mathematical thinking, and a productive disposition" (Baroody, 2004-a). We need to find ways to help children connect their informal experiences to school math.

Before the constructivist revolution in education, drill and practice was a major focus of formal education. People used educational approaches based on a transmission model for learning – I know something that I want you to know, so I tell you and then you know. It might be a little scary using an investigative or exploratory approach where we can't just tell children about the connections between school math and their informal math, but we don't have to let their investigations be haphazard or unplanned (NCTM, p. 75). Instead, we can use activities that are specifically designed to be problematic for children at a variety of conceptual levels so that their current knowledge can be put to the test. Disequilibrium, conflict, and problem solving are required for the kind of learning that results in the ability to solve problems in a wide variety of situations (Yackel, Cobb, & Wood, 1991). Steffe (2004) regards the mathematics of children as "a mathematics only children can bring to life through their interactions," and agrees with Kieran that adults provide occasions for bringing forth, sustaining, and modifying the mathematics of children. Teaching strategies and imposing their use might not be the best idea (Steffe, 2000), but Baroody tells us that recent research suggests a way that teachers can *guide* children's *invention* of procedures – one that involves helping students make the conceptual breakthrough needed to understand the procedure (2004-a). Baroody champions a "standards-based investigative approach" – we want them to learn some useful strategies, but we don't want learners to use them blindly or inappropriately.

Notice that this follows nicely with the suggestions from designing constructionist tool kits. We need to keep in mind the personal and epistemological connections children are likely be able to leverage in our tool kit. The literature on learning through design, expressive mathematics, and math learning highlights the need to set up software to allow for meaningful interactions and further suggests that we structure activities to help learners take advantage of the affordances of the environment. To set up appropriate activities, we need to know more about children's common misconceptions and ways to help them build appropriate representations and invent useful procedures for dealing with math. In this research, I chose to focus on fractions and symmetry as targeted conceptual areas. There seems to be a lot more information about children's understanding of rational numbers than there is about symmetry, but I will present some of the work from each that was most influential to mine.

Fractions

Part of the reason rational numbers (including fractions) are so hard for students to understand is that there are so many meanings associated with them (see Behr et al., 1992 for an in depth discussion about differing views). Research from the Rational Number Project (RNP) (e.g. Behr, Lesh, Post, & Silver, 1983; Behr, Wachsmuth, Post, and Lesh, 1984) and others have documented common errors made by students when operating on symbolic representations of fractions. Some of this research suggests that learners' understandings of fractions are rooted in using rote procedures (often incorrectly) rather than in basic concepts underlying the procedures. Kieren (1976) identifies five rational number sub-constructs: part-whole, measure, ratio, decimal, and operator. Even within one sub-construct of rational number, learners must deal with translation within and between modes – including pictorial, verbal, symbolic, and real-world (Behr et al., 1992). Understanding fractions is a matter of understanding different representations, how they relate to other representations, and how to translate fractions both within and between representations (Hiebert & Carpenter, 1992).

Helping students learn fractions, then, should involve helping them see connections and relationships between different representations so that they understand the underlying concepts as well as the procedures that can be taken to operate on the different representations and translate between them. Behr et al. emphasize that the translation issue should not be taken lightly since previous work suggests that the gap between manipulative aids and symbols is significant, and the mental bridge needed to cross the gap is complex (1992).

Much of children's existing knowledge about fractions is explored through asking them to complete "fair-sharing" problems (e.g. Baroody with Coslick, 1998, Hunting and Davis, 1991, Mack 1990, 1993). Lamon (1996) did extensive analysis of partitioning strategies children used for fair-sharing problems. She emphasized the importance of the graphical nature of the activities the children completed (not just symbolic) for helping them refine their strategies for equal partitioning and effectively describe how much is in each share and how the share relates to the total amount to be partitioned. Baroody (2004b) suggests that,

> "A class discussion could make explicit the following fundamental fractions concepts:
>
> 1) The shares must be fair or equal in size (fractions involve a special situation where all the parts of a whole are equal in size)
>
> 2) Three halves means "three one-halves" (fractions embody multiplicative reasoning)
>
> 3) Three halves and one and a half represent the same amount (a fractional amount can have different names – which we call equivalent fractions)"

One notion that has been widely adopted is that fractions lack an intuitive developmental foundation that counting numbers have (Dehaene, Gelman, and Resnick as cited by Sophian, 2000). Some work seems to suggest that it is not lack of intuitive foundation that prevents meaningful learning of fractions, but simply that a different approach (like instruction that revolved around children's solutions to fair-sharing problems) would work better (Empson, 1999). Mack (1995) found that children often treated fractional units (regardless of their value in relation to the whole) as though they were wholes. Lamon's (1996) work echoed that same concern – that students were unable to understand the relationship between the whole and the fraction.

Sophian suggests that learners might have an easier time with spatial ratios (where the fraction is represented by a certain portion of the area of something having some property) because they, "provide a way of conceptualizing the quantitative value of the ratio as an integral whole and as a means of identifying the equivalence of fractions that are numerically different but proportionally the same (such as 2/3 and 4/6)" (Sophian, 2000). One source of confusion in understanding fractions may be confounding knowledge about whole numbers that is already in place (Sophian, 2004). For example, children are used to the idea that 3 is greater than 2, but in the denominator 3 might make a smaller ratio than 2. Thus, fractions behave in a way that might conflict with established knowledge about whole numbers. In one study, even pre-schoolers and kindergarteners successfully made judgments about spatial proportionality (Sophian, 2000). In a similar study with 5, 7, and 10 year olds (Sophian & Yamashita, 2000 as cited in Sophian, 2004), the availability of numeric representations so threw off kindergarteners thinking that it disrupted their proportional comparison. However, the numeric information was quite useful to the 10-year-olds. Sophian suggests that this is the result of "overly restricting the range of numerical activities and relations typically presented to young children." Mack (1995) found that children tended to overgeneralize both whole number and fraction knowledge to the other domain as they attempted to construct meaning for symbolic representations of fractions.

In DigiQuilt, students learn by directly manipulating spatial ratios, meaning that they make changes to the area of a quilt block that is covered with any given color by placing tiles in a constrained area. In spite of Sophian's uncertainty about how far the use of spatial ratios will take children toward understanding numeric ones (2000), I provide the learner with numeric feedback in the hopes that the presence of the feedback will solidify the connection for learners (whether by drawing their attention to unexpected mismatches between the spatial ratios and numeric feedback or simply showing the two in conjunction with each other).

Research on the use of multiple representations for math learning suggests that a combination of linked visual, verbal, and symbolic representations helps learners (Kaput, 1986, 1992). Moreno and Mayer studied the use of multimedia environments for math learning (1999). In their work, they lay out two seemingly competing theories – multiple representations vs. cognitive load. Their results suggest that although multiple representations would seem to add to the cognitive load in some students, those who were already high achievers benefited from the use of multiple representations while students who were not high achievers did not seem at all hindered by the extra information (ibid.). From pretest to posttest, the high achievers showed the highest gains on the most difficult problems, reduced the number of conceptual bugs in their strategies, and learned faster during the training (ibid.). This result is consistent with Sophian's work with learners of a similar age (Moreno and Mayer worked with 6^{th} grade students).

In addition to learning more about how children learn about fractions and common mistakes children make, I used ideas and interview questions from the Rational Number

Project (RNP, 1979-2002). These interviews helped me see children's thoughts in action and helped me plan activities that I thought would best lead to conceptual change.

Symmetry

The literature in mathematics education is less informative about symmetry than fractions as far as describing what exactly is hard about learning it. Though there is surely more information today than there was in 1970 when Dodwell asked if there was not more to symmetry than, "a simple perceptual phenomenon," not a lot of focus had been placed on understanding both the perception and cognition involved in understanding symmetry (Dodwell, 1971). In fact, most of the literature I could find about symmetry dealt with the perception of symmetry. Much research has been dedicated to finding out more about how people perceive symmetry and developing theories about why some kinds of symmetry seem easier to detect (Palmer & Hemenway, 1978; Royer, 1981; Wenderoth, 1994; Pashler, 1990; Wagemans, 1995; Bruce & Morgan, 1975; Freyd & Tversky, 1984).

What we do seem to know about symmetry is that bilateral symmetry is the easiest for people to identify. For line symmetry, a vertical line is easiest, followed by horizontal, and finally diagonal (Palmer & Hemenway, 1978; Royer, 1981). Maria Klaus-Boelte and John Kraus further suggest that it is more difficult for the child to create diagonal forms of symmetry (1882, p 97). Still, detection and construction are two different skills. Freyd and Tversky (1984) report that participants in their study were likely to remember "almost" symmetric images as being symmetric. They use the terms local and global to refer to different levels of features of an image, saying that the perceiver grasps the overall shape (global) and then attends to details (local) to varying degrees. Perception of global symmetry might cause the perceiver to overlook local violations of symmetry (Freyd & Tversky, 1984). The distance of objects from the axis in mirror symmetry also impacts the likelihood of the perceiver noticing violations of symmetry (Bruce &

Morgan, 1975). For learners, then, it would seem that we would want to help learners in their symmetry construction and detection by giving them points of reference for both construction and detection of symmetry. Pashler (1990) suggested that cuing subjects about the location and orientation of the axis of symmetry helped them perceive it with greater speed and accuracy. The presence of a line imposed on the designs the children are creating, then, might be useful both for helping children detect symmetry and for helping them share their designs with others.

Valenzeno et al. found that teachers' gestures can facilitate students' learning about symmetry (2003). In another study, students who used gesture received more varied help from their teachers since the gesture helped the teacher elicit more nuanced understandings of the learners (Goldin-Meadow & Singer, 2003). When teachers' gestures suggested strategies that differed from the verbal information being provided, students made greater gains in understanding (Singer & Goldin-Meadow, 2005). It seems, then, that learner can benefit from opportunities to use gesture in relation to solving problems. In the classroom, when I talked about symmetry, I often referred to the quilt blocks folding and asked where pieces would "land" if they were folded over a certain line. I used gesture in my explanations to give the learners a way to visualize what I was asking. The digital quilt blocks and select-a-grid tool were meant to help facilitate discussion about both fractions and symmetry by giving the learner something to talk about and reference in those discussions.

Manipulatives

Research in math education (e.g., Clements & McMillen, 1996; Clements & Battista, 2000; Hiebert & Carpenter, 1992) and many math curricula suggest that working with manipulatives can help learners connect the concrete materials that are actually manipulated with the (often more difficult to grasp) symbolic or abstract ideas that students need to learn. Manipulatives have been used in math classrooms for quite some time as a way of helping students transition from concrete, physical objects to the abstract, symbolic language of math.

Some history on manipulatives

Froebel (1826/1887) developed a set of "gifts" that were meant to allow children to explore how the world works through guided play with them. Each gift was designed with a particular set of activities (occupations) in mind (1861/1899) with the overall goal of educating the pupil through self-activity. Later, Montessori developed a whole approach to education that centered on providing meaningful objects in a prepared environment as a way to learn (1909/1912). This type of structured environment was believed to foster the kinds of experiences that lead to learning. Since freedom of the child was considered of the utmost importance, teachers were cautioned that lessons should be concise, simple, and objective (ibid., p 107-118) if they were to be given at all. Both Froebel and Montessori are often cited as pioneers in the use and development of manipulatives.

Manipulatives are not magic

Although they can be useful learning tools, manipulatives are not magic (Ball, 1992). Not all research on the use of concrete materials in the classroom has yielded exceptionally positive results (Sowell, 1989). Moyer (2001) reported that teachers in their study were unsuccessful in using manipulatives to help children *learn* math. Rather, the teachers viewed the use of manipulatives as a way to reinforce previously learned content or to just have "fun." Research suggests that just using the manipulatives is not enough to promote deep learning. Some research (e.g., Resnick & Omanson, 1987; Fuson & Briars, 1990) suggests that when learners simply follow a prescription for how to work with the manipulatives, they are not experimenting or getting a sense of the tensions that exist between what is in their minds and how they can express it. Hiebert and Weane (1992) found that when manipulatives were used in this manner, little to no learning occurred. So, just using concrete materials is not enough, and even using manipulatives in prescribed ways is not enough. It is important to think no only about how the learner will use the materials, but in particular how they will be able to come to understand abstract ideas through their explorations. Comments and attitudes that imply that manipulatives are "just for fun" can undermine the utility of manipulatives for learning meaningful mathematical concepts (Moyer, 2001, p 191). The design of the manipulative, how the manipulative is used, and the social context of use all contribute to the success of using manipulatives for learning.

Design of manipulatives

"Good manipulatives are those that aid students in building, strengthening, and connecting various representations of mathematical ideas" (Clements, 1999).

Research on physical versus virtual, computer-based manipulatives suggests that both can offer connections between the concrete and abstract when used with proper guidance, but that learners often gain *more* from the computer-based manipulatives (or from using both) when other factors are the same (Thompson & Thompson, 1990; Clements, 1999). It is hypothesized that this increase in performance is due to the fact that the computer can provide a world with more constraints that keep relationships consistent. One of the most important outcomes of keeping these relationships constant is that when a learner makes a change to one representation and one of the related representations behaves in an unexpected way, the learner's attention is drawn to this expectation failure (Thompson, 1992). In the Thompson study, some learners were unable to reconcile this disequilibrium in the time available and, therefore, did not perform better on tests as a result of using the computer-based manipulative. Still, this disequilibrium is an important part of the learning process that is less likely to occur without the updating representation drawing the learner's attention to a surprising result – expectation failure will not occur unless the learner notices that the expectation was not met. Clements & Battista (1992, p 449) suggest that the difference between concrete and "nonconcrete" may not be as important as manipulability.

<u>Trends in manipulatives: differences in physicality – tradeoffs</u>

As mentioned in the introduction of this thesis, manipulatives have begun to transcend their physicality in the form of virtual manipulatives. These virtual manipulatives are screen-based and often very similar to traditional physical manipulatives in all aspects except their (lack of) physicality. In addition to these virtual manipulatives, some researchers are exploring "digital manipulatives," which are physical manipulatives with computational power embedded inside (Resnick et al., 1998). Resnick and his colleagues are most interested in using this new kind of manipulative to expand the kinds of topics that can be effectively explored through the use of manipulatives, so that younger children can learn about concepts related to dynamics and systems (ibid.). More recently, Zuckerman et al. (2005) described a set of blocks designed to help children learn about systems.

This is not the first time researchers have encountered the dilemma of physical vs. virtual world for math learning. For the LOGO turtle, the journey from the physical world to the microworld meant added accessibility in the classroom, but perhaps a less obvious form of body syntonicity for which it is so famous. Imagining the turtle from different perspectives probably made more sense when there was more reason to do so. On the computer, stepping into the turtle's shell is less necessary since there is a more universal idea of "up". In the physical world, if the turtle is in the middle of a circle of children, then controlling the turtle in terms of its orientation (and imagining yourself as the turtle) has greater importance. Body syntonicity plays a much smaller role in DigiQuilt than in LOGO, so I would argue that the tradeoff from physical to virtual is much simpler for DigiQuilt since there was less to lose. The virtual world offers so many more affordances for connecting the concrete and symbolic and for helping children see those connections that it just makes sense to move to the computer.

Classifying manipulatives

In addition to classifying manipulatives based solely on their physicality, some researchers have begun to classify manipulatives based on their inspiration. In their 2005 paper, Zuckerman et al. define two kinds of manipulatives: those inspired by Montessori (MiMs), and those inspired by Froebel (FiMs) – they focus on the MiMs. Their claim is that FiMs are design materials, fostering modeling of real-world structures, while MiMs foster modeling of more abstract structures. According to their description, DigiQuilt should be classified as a FiM, which makes sense since design is a focus I intended. However, I am not certain that this classification would capture the full design of DigiQuilt since it aims to foster both design and connections to symbolic mathematics. I am also not certain that this classification addresses the social aspects of a socio-technical system – scholars of Froebel seem to suggest a focus on the activity structure that is not at all clear in this classification (where building blocks, K'nex, LEGO bricks, LEGO Mindstorms, Tinkertoys, and Zome are all listed as FiMs). I would imagine that pattern blocks and quilting tiles would be considered FiMs as well, but I think that the context of their use could shift their focus toward the symbolic end of things. Perhaps there is a continuum that is inspired by this system of classification that would allow for a manipulative/environment hybrid to be classified somewhere in between.

Using manipulatives

Helping teachers use concrete materials as not just a way to keep students attentive and interested, but as a way to help students learn to *model* mathematical concepts and to *understand* links between mathematics and the students' own thinking is complicated (Ball, 1992), but important. In physical environments, where the computer does not manage relationships between concrete and abstract representations, learners can make intuitive actions and corrections without explicit awareness (Clements & Battista, 2000).

Getting feedback from the computer allows them to make and correct mistakes without showing them to someone and to see immediate effects of their actions (Kaput, 1992). This independent success can serve to increase a learner's self-efficacy, which can in turn motivate students to use better cognitive strategies and achieve more (Schunk, 1983, 1991, Schunk & Swartz, 1993, Pintrich & Schunk, 1996). Based on this, I decided to provide feedback about fractions so that DigiQuilt would allow students to check if their designs meet challenges without asking for help. Although connections on the computer screen do not map directly to connections in children's minds (Ball, 1992), shifting the focus to *understanding* the relationships rather than maintaining them is a good start (Kaput, 1992). Though students may be frustrated that the computer environment uncovers some of their misconceptions, this can lead to better development of mathematical abilities (Clements & Battista, 2000).

Context of use

The social context in which materials are used may account, in part, for their effectiveness (or ineffectiveness) in helping students understand (Hiebert & Carpenter, 1992). I envisioned small group and whole class interactions supporting students as they use DigiQuilt to construct artifacts and attempt to understand how different solutions can apply to the same challenge. Concrete materials provide both a public presentation to which attention can be drawn and a focus for discussion (Hiebert & Carpenter, 1992).

Research on small group interactions as a source of learning opportunities in mathematics (Yackel, Cobb, and Wood, 1991) showed that collaborative dialogue and resolution of conflicting points of view along with the use of activities that are designed to be problematic in specific ways are crucial features of a cooperative learning environment that relies on intrinsic motivation. DigiQuilt has affordances for being a catalyst for classroom discussion by providing opportunities for artifact-centered discussions with tools and activities to point learners to particular features of the artifacts. Indeed, enhancing discourse is at least one of the purposes of using a manipulative in the classroom. When students verbalize strategies for solving problems it helps them attend to important features of what they are doing and remember that strategy later (Schunk, 1995). So, getting the learners to talk about something as they are learning about it is a worthwhile goal.

As part of the enactment of my research with DigiQuilt, I used challenges to guide learners to difficult territory and draw their attention to solving problems that could potentially lead to deeper understanding of tricky concepts. I told students to feel free to talk to their neighbors about problems they were solving and solutions they found. I encouraged them to talk through the problems, and when they had questions, I tried to help them come up with ways to solve the problems.

Support and surprise through tools and challenges

"Transparency" is used to describe the extent to which a tool is used without requiring thought about the tool itself (Lave & Wenger, 1991). Meira (1998) talks about the transparency of a tool not as an attribute of the tool but as something that we can only talk about with respect to its use by a specific person or group. So, if it is easy for me to use DigiQuilt, then I can focus on the things I am trying to learn rather than on the software, but if it is hard for me to use the software, then it is not acting transparently as a tool. DigiQuilt might not act transparently as a tool for all learners, but the kinds of things that are tricky are meant to bring difficulties to the foreground. Perhaps more clearly stated, when something is difficult in DigiQuilt, it is through the development of an understanding of the use of the "tool" that an understanding of the embodied concept is gained. Meira sates that although transparency is not an attribute of a tool per se, that is not to say that it is unimportant to analyze a tool for its affordances, rather that it is *as* important or even more important to think about the transparency of a tool to a specific person in a specific situation. I think this transparency can be likened to fading of scaffolding in some software systems – as a tool becomes more transparent, and concepts become understood to the extent that the tool is used to get something done rather than as a part of the learning itself, the support is not needed, the tool itself is not attended to, and it fades into the background.

In his article, "Does Easy Do It?" Papert (1998) tells us that kids enjoy "hard fun" more than things that are easy. In fact, I think that this relates closely to the concept of Learner-Centered Design (Soloway et al., 1994) vs. User-Centered Design (Norman & Draper, 1986) in that the goal is not to make things easy, but rather to make them visible or salient. In the design of DigiQuilt, it is not my aim to make everything easy. In contrast, since my goal is to help children understand fractions and symmetry, it is more important for me to help them see and address any misconceptions they may have about those topics. To do that, I focus on bringing difficulties to the foreground through the use of challenges, and helping learners understand the challenges through the use of mathematical tools. But, having useful technical tools is probably not enough to ensure conceptual change. Pintrich, Marx, and Boyle (1993) suggest that for conceptual change to occur, we may need to know more about the impact of classroom contextual factors and motivational beliefs of learners. Since design is seen as motivating for students, I aim to support learners' design efforts – I provide some useful design tools that allow them to easily execute certain design-related actions. By providing support for design activities, I hope students will become engaged in designing personally meaningful quilt blocks. Because they can design personally meaningful quilt blocks, they can explore the quilt blocks using mathematical tools I provide, and their designs are created so that they solve challenges that bring difficult concepts to the forefront, I believe I have created a situation where children can learn in a "hard fun" environment.

As stated in the introduction to this dissertation, my hypotheses stem from the idea that combining "school math" and art through quilting with a computational manipulative that supports design will allow us to leverage both the affordances of using manipulatives to learn math and the affordances of learning through design. Research on learning through design tells us that a design approach engages learners. The math literature suggests that manipulatives can be an effective way to help learners connect the concrete with the abstract. To show that combining math and art through designing quilts allows us to leverage the both affordances of manipulatives and of design, I will need to show two things:

1. Children engage with the manipulative in an expressive way to design things they care about. The design environment will motivate them to the extent that they can persist through difficulties to solve challenges (some that are given to them, and some that they add for themselves).
2. The children connect the concrete quilt block designs they create with the abstract "school math". As a result of working within the socio-technical system I have

> designed, students will find the support they need to draw connections between the abstract and concrete. We can help children mathematize their worlds.

In this chapter, I outlined previous research and work that informed the design of the DigiQuilt socio-technical system. It is my goal to support students as they begin to understand their world in a mathematical way. To do that, I have provided challenges that will bring targeted concepts to the foreground, software tools that facilitate discussion with peers and help learners make sense of surprising results, and an expressive, artistic medium that I think students will find motivating, In the next chapter, I will describe the design of DigiQuilt in the context of two formative studies, highlighting the design and development of the various tools in the technical part of the system. I will describe how the learners' experiences with the DigiQuilt software influenced the design over time. The goal was to help children mathematize their worlds, and the focus of the next chapter is the design of the software part of the socio-technical system that helped them do that.

CHAPTER 3

DIGIQUILT

DigiQuilt (Figure 4) is a virtual manipulative environment for children to learn about fractions and symmetry by designing patchwork quilt blocks (Lamberty & Kolodner, 2002). Students from third to fifth grade at three schools in two public school systems have used DigiQuilt in several different forms throughout its design history. Their experiences have shaped the design of the system, particularly the addition of tools to help students learn through the process of creating and examining their designs. In this chapter, I describe the design history of DigiQuilt from its early days through two years of development and two formative studies. I explain how the experiences of the students participating in these studies, coupled with previous research on learning and educational technology, informed the design of a new kind of manipulative environment with explicit support for both design and math. I conclude this chapter with a description of DigiQuilt as it was at the start of my dissertation study, particularly focusing on its affordances for design and math learning.

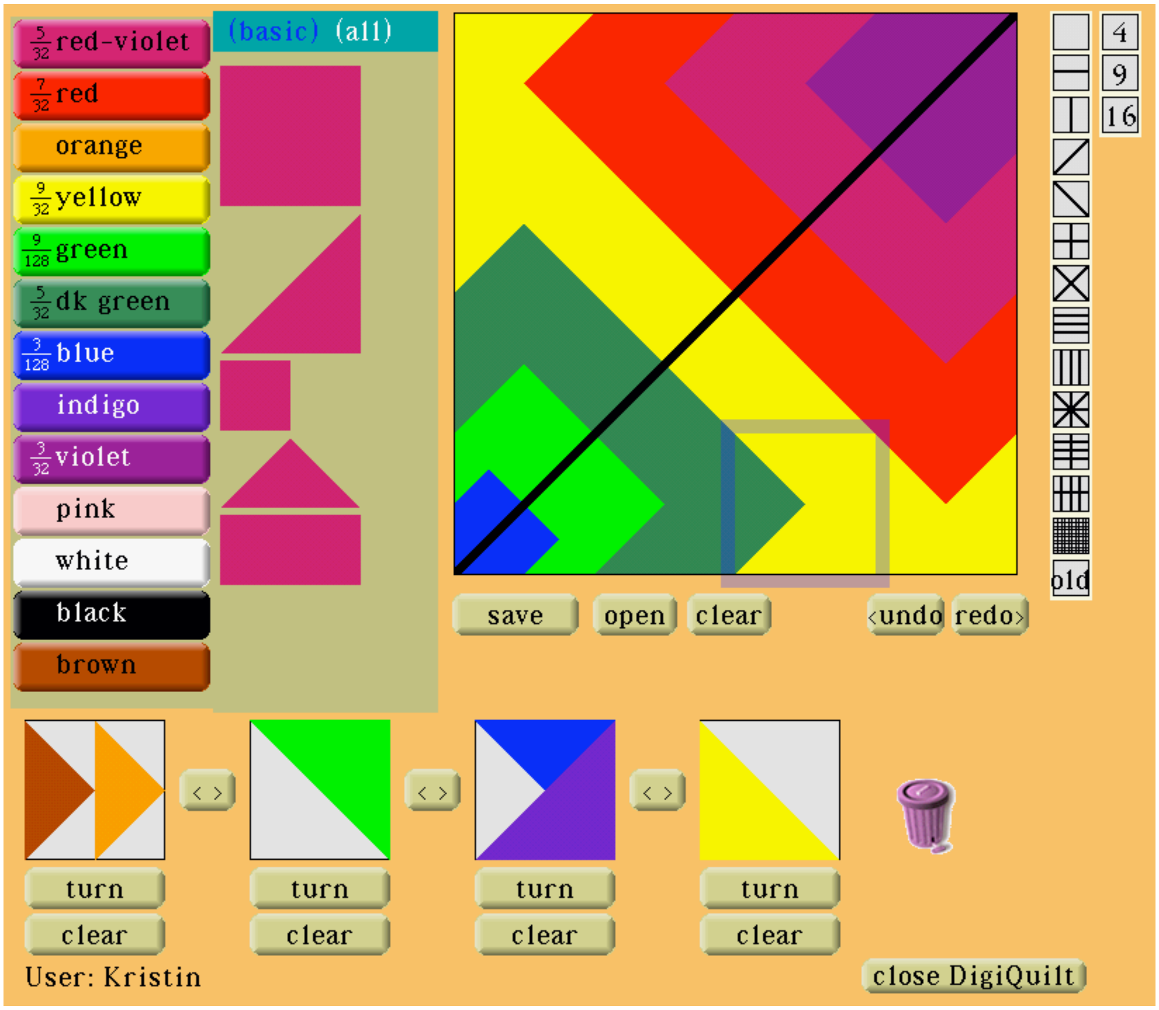


Figure 4. A screenshot of DigiQuilt.

Getting Started: From Quilting to Paper Pieces

Beginning from the idea that patchwork quilting had some interesting affordances for learning, I needed to develop a system of social and technical supports – a socio-technical system (Herrmann & Loser, 1999)– that would allow learners to leverage their personal connections to learn about the targeted content *through* their design activities. Quilting offers many possible learning connections to math and art, and several opportunities for personal connections for students because it is a craft that has been practiced in many cultures throughout history. The question was, initially, how could I create a learning environment that leverages some of these possible connections?

As described in the background chapter, research on constructionism (e.g., Papert, 1991, 1980; Resnick et al., 1996) and learning through design (e.g., Kolodner et al., 1998, 2003; Shaffer, 1997) suggested that a design environment would be engaging for learners, and that it would provide the kinds of experiences and connections that make for a good learning environment. The math literature suggested that manipulatives could help learners connect the concrete and abstract representations of mathematical concepts (e.g. Behr et al., 1983). Combining the constructionist approach to education and the suggestions about how to use manipulatives (for exploration rather than in a prescribed manner) led me to imagine a manipulative with affordances for both math learning and design. Since some tessellating patchwork quilt patterns reminded me of working with pattern tile manipulatives in elementary school, the quest to imagine a way for children to design patchwork quilts using a manipulative began.[3]

First, I tried making felt quilt patches since they can be rearranged like traditional manipulatives, and felt is a fabric, which made sense for quilting. Though it was reusable, I decided that it would not lend itself well to making lasting constructions in the classroom – it was too expensive to allow students to make and keep lots of designs. Because it is readily available, relatively inexpensive, often found in classrooms, and a familiar material for art projects, construction paper seemed to be a more appropriate choice of material.

Froebel, the inventor of kindergarten, developed a series of manipulatives – now commonly called "Froebel's Gifts." The blocks in Froebel's Gifts (Froebel, 1899)

[3] There are, in fact, several different kinds of "quilting tile" manipulatives available on the market today, though some fairly thorough searches for some in 2000 and 2001 were fruitless.

inspired the initial shapes considered for the system. In my system of paper pieces, the proportions of the pieces in relation to each other echoed the proportions of the shapes found in Froebel's gifts – particularly the 3rd, 4th, 5th, 6th, and 7th gifts. Gifts 3-6 are each sets of small blocks that can be assembled to make a larger cube – cubes that form a cube, half-cube-triangular prisms that form small cubes that form a cube, etc. The 7th gift is the first of a series of tiles (much like pattern tiles). In playing with Froebel's Gifts, I noticed it was easy to fit the blocks from the Gifts together in an aesthetically pleasing way. Since I wanted to set up the learners with as much opportunity as possible for designing attractive quilt blocks, starting with squares and half-square triangles made sense. To provide guidance for how to place the patches to form a quilt block, I created a 16-patch base block – essentially, specialized graph paper with 64 small squares and darker lines outlining the 16 patch-sized squares that would form a complete quilt block (see Figure 5). I started using a 16-patch base block for the quilt blocks so that there would be ample space for learners to create a huge variety of designs, but still enough constraint to keep the designs simple enough to talk about them in terms of fractions and symmetry.

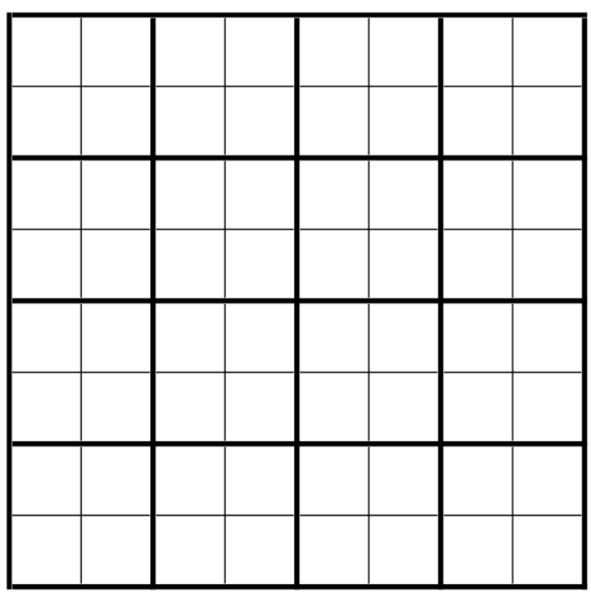


Figure 5. The grid from the paper version of the 16-patch base block.

For some initial user testing, I set up a system of paper pieces and grids for making designs. I cut out construction-paper squares and half-square triangles in about a dozen colors. I provided the learners with the paper pieces, glue and glue sticks, and 8.5 by 11 inch sheets of computer paper with 16-patch base blocks printed on them. I began by having colleagues, and then third grade children in a local school, work with the paper pieces in the framework of the 16-patch base block. The activities were challenge-driven in that users were asked to complete challenges like, “Make a quilt block that is 1/2 one color and 1/2 some other color.” This challenge-driven approach allowed learners to arrange their quilting tiles (paper pieces) in any way they chose within the framework, and did not specify any particular way to address the challenge. The challenges were chosen with particular learning goals in mind, and it was the role of the facilitator to help children recover from expectation failure in a meaningful way.

Results

When working with paper pieces and grids, the learners sat in a circle on the floor or around a table with sandwich-bags of paper pieces in the middle. The children tended to select two or three colors of shapes to work with and bring those bags of shapes closer to themselves. I assigned a challenge and then everyone worked on solving that challenge. Once I could see that most children had completed a design that solved the challenge (usually some had completed two or three designs by that time), I asked the children to choose one design to share and we would go around the circle sharing the designs. For example, a student might say, “My design is ½ blue and ½ yellow. I know it has equal amounts of blue and yellow because each time I added a blue piece, I added a yellow piece of the same size.” Alternatively, if the student did not offer an explanation of how they knew it solved the challenge, I might ask the other students if they agreed that the design solved the challenge.

In trying this paper prototype with colleagues and then with third grade students at a local elementary school, I found that users generally knew how to put the shapes onto the grid in a way to cover the area and solve some challenges. The learners could share their designs with each other and describe the designs in terms of the challenge at hand. If the learners needed scaffolding to participate successfully, I was able to help.

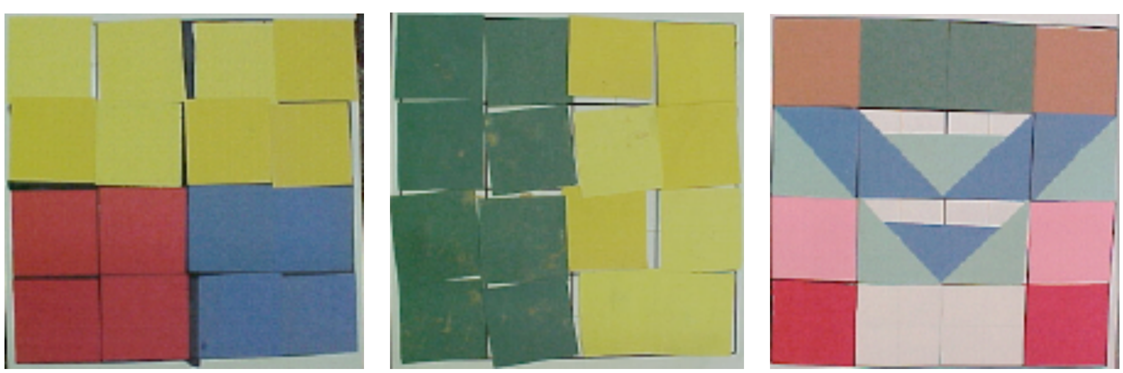
In general, I liked this format for talking about designs because it gave the children a chance to not only share their designs, but to see several other designs that were different from their own and still solved the same challenge. Previous research suggests that showing non-examples and comparing a range of similar examples helps focus the learners' attention on certain aspects of the specimens and prompts discussion (e.g. Clements, 2004, page 39). However, several difficulties arose: keeping the shapes from moving around, aligning the shapes with each other and the grid, and making the designs quickly after deciding on an idea. Not all children were equally able to create beautiful designs that were easy to talk about mathematically due to these difficulties.

Dexterity and physical ability

One limitation on children's design experiences seemed to be dexterity – some students had a difficult time working with the pieces of paper. Simply gluing the paper pieces to the base grids that were provided was quite difficult for some children, and they completed this task with varying levels of success (see Figure 6 for some samples of paper-pieced designs). Some students had an especially difficult time making their designs look "nice." Lumps of glue, misaligned shapes, and wrinkly quilt blocks were not uncommon. Figure 6b is one such example of a paper quilt block – notice the gaps between patches and glue (from a glue stick) on top of most of the patches. Those students who were successful in achieving a tidy result often took a lot of time to glue their pieces down.

Aligning shapes within the grid

Some children overlapped shapes or tried to place shapes in ways that were inconsistent with how the grid was designed. Some students would place pieces that were half the size of the big squares (half-square triangles) so that they overlapped two different grid-squares (see Figure 6c). Placing half-square triangles so that the corners fit within a grid space seemed difficult for some children, so they placed the triangles with the hypotenuse where a side should have been. The spaces that are left unfilled in either of these cases are more difficult to measure since they do not correspond directly to a particular piece that is available to add to the quilt block. These difficulties with configuring the shapes within the grid led to designs that did not lend themselves well to talking about the math of the designs.



(a) **(b)** **(c)**

Figure 6. Students' designs made using paper pieces.

Supporting understanding of equivalent fractions

It is notable that even with early work using paper pieces, with some guidance, children were able to gain insights about equivalent fractions. When asked on a pre-test to place the correct symbol (<, >, =) between 1/2 and 2/4, only two answered correctly. This suggested that the students were not making the connection between the concrete and abstract representations of the fractions and symbols: here was an opportunity to connect the manipulatives to solving a problem. To help them see another way to look at the problems, I moved the shapes (grouping them as shown in Figure 7) and described what I was doing. The students looked confused, but after I moved the shapes a couple more times one student said, "Oooooh. I get it. It's the same *amount* so it's the same *fraction*!" Her tacit knowledge about the equality of the area covered by the squares was thus connected to the idea that the two different formal representations were equal. By leading the other students through her thought process, she shared that connection. It seemed as though seeing the two quilt designs and the *transformation* of the configuration of the squares that comprised the design helped her see that these two fractions were the same "amount."

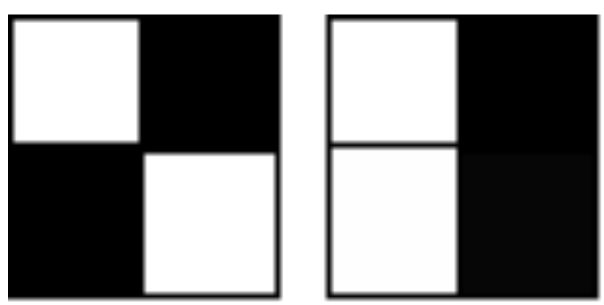


Figure 7. Two arrangements of 1/2 (one that emphasizes 2/4 and one that emphasizes 1/2).

Though using physical manipulatives is nothing new in elementary school mathematics learning, this supported the idea that students could learn something by comparing quilt designs and talking about the fractions they represent. In addition, this event provided a clue for me that helping learners *see* the same shapes in a different arrangement and *realizing* that these are the same shapes (that the only thing that is different is the arrangement) could help them grasp the idea that two symbolic fractions have the same value. I learned that I needed to help learners not only view the quilts in a different way, but to help them realize that they were doing just that. This is not all that different from the role of the teacher in interactions suggested by Froebel (1861/1899).

Sharing quilts and mathematical ideas

In addition to being useful as shared representations of mathematical ideas, the designs that the children made could be displayed in the classroom or taken home. We made a class quilt with the school colors to display in the hallway – each quilt block was (supposed to be) half blue and half yellow (Figure 8). I explained how the whole quilt was half blue and half yellow as a result (though looking back, it would have made more sense to help the students discover that with some guidance). Still, the paper quilt blocks could be displayed in the hall for everyone to see, and the students and teacher both seemed to like that idea.

Figure 8. Classroom quilt made by 3rd graders with paper pieces on display in the school's hallway.

Moving On: From Paper Prototype to the DigiQuilt Software Environment

For the first version of a software-based environment for designing patchwork quilt blocks, I took into account difficulties that the learners had using paper pieces and grids, as well as affordances the medium of quilting could offer. Since I wanted the learners to design public artifacts, I wanted to help them make those artifacts easy to discuss (in order to encourage a dialogue between the learners). Also, when students are proud of how their artifacts look, I thought they would be more likely to want to share their artifacts with a wider audience. Moving from paper pieces and glue to a virtual manipulative would enable children to consistently make their designs look "good." During this phase, my main concerns were to help users manipulate their pieces in productive ways, to make the design process simple enough that even less-dexterous users would be able to make their designs look good, and to promote the construction of designs they could talk about mathematically.

Design

I designed the software to limit how shapes could be manipulated and placed into the patch-holders. In this first version of DigiQuilt (see Figure 9), shapes snap into patch location depending on the location of the cursor. This alignment is preserved even when the shapes are moved into different patches (this allows users to move groups of pieces from one patch-holder to another). Besides freeing users from needing to worry about lining up their pieces perfectly for the design to look nice, I limited how the pieces could be rotated – 90 degrees with each click of the button (the “turn patch” button near the lower-left corner of the screen). This limitation insured that users could not accidentally place their pieces so that parts of them overlapped patch lines. Kaput (1992) talked about constraints and supports (“CS” structures) in virtual manipulatives as one in the same since both constraints and supports work to help children learn. In the paper system, I provided the grid as a constraint, but there was no way to limit how the learners could actually place their paper pieces on the grid. Designs made with DigiQuilt are more mathematically predictable than designs made using paper piecing due to these constraints. Since one purpose of DigiQuilt is to support learners’ creation of artifacts to discuss, it is important that the designs be as mathematically understandable as possible for the learners.

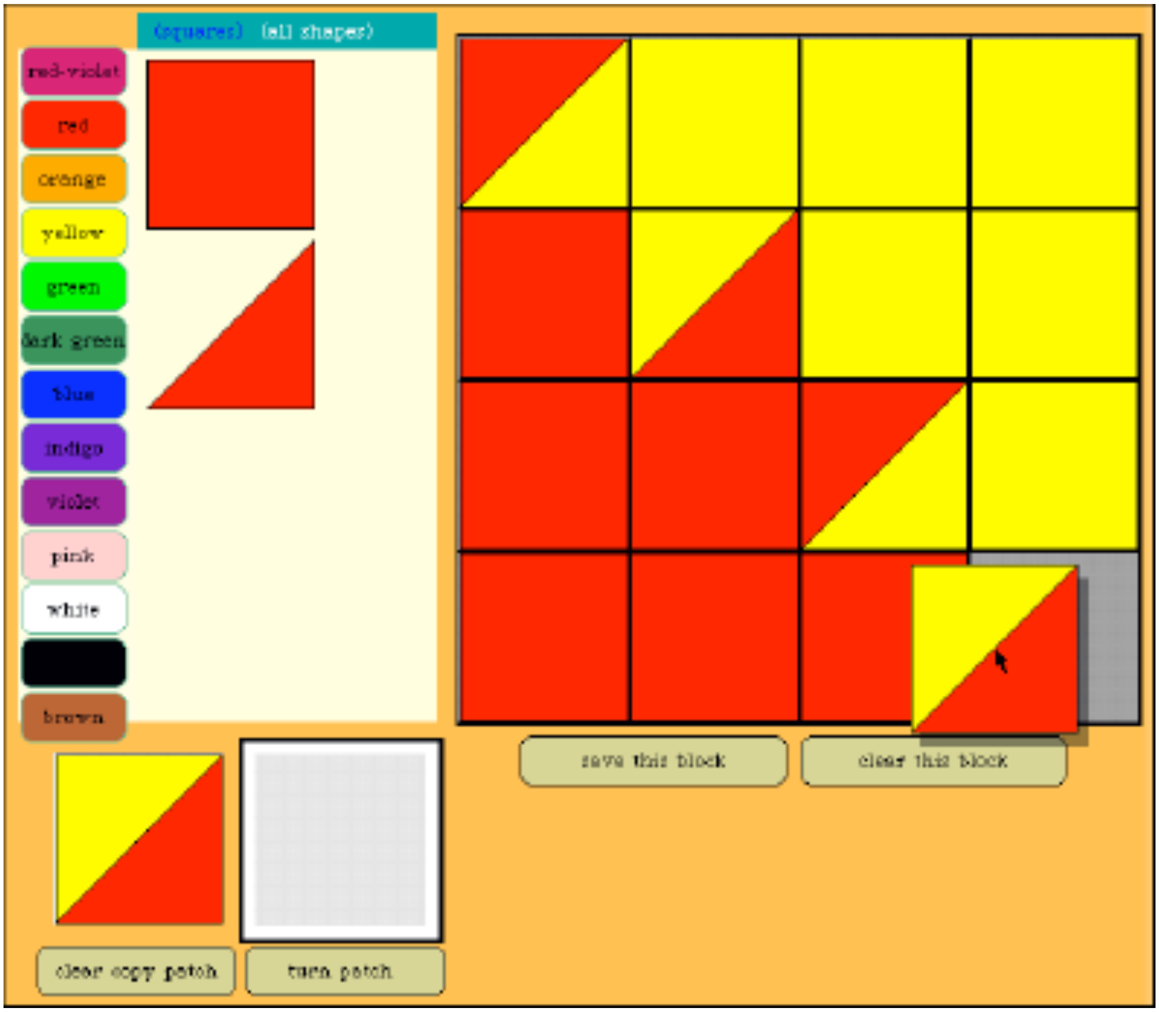


Figure 9. An early version of the DigiQuilt software (actual third grader's design featured).

Generally speaking, the interface of DigiQuilt is a form of direct-manipulation. Learners use a point-and-click interaction style (as opposed to drag-and-drop) to select and move pieces from place to place. This style of interaction has been shown to be faster, less prone to errors, and preferred by children (Inkpen, 2001). The tools that were initially made available to manipulate shapes in DigiQuilt were the facilities for rotating pieces or patches (combinations of pieces that fit within a patch-holder) and the ability to make copies of patches (Figure 9, lower left corner).

The rotation of patches and pieces in DigiQuilt was designed specifically to afford the learning of a listed national standard for third graders: the ability to recognize the same shape in a different orientation (NCTM, 2000). By presenting each shape on the palette in only one orientation and requiring students to rotate them to fit their needs and solve the challenge, the software afforded learning to recognize the same shape turned in different ways. With a physical manipulative, where shapes are scattered on the floor or table, it is possible to select a shape that is already in the correct orientation without this explicit awareness. The manipulation of the shapes becomes more deliberate in this type of virtual environment (Sarama, 2004, page 365).

The tool for duplicating patches (the "copy patch") was included to allow users the ability to make repetitive designs quickly. Many patchwork quilt designs used by quilters in the "real world" involve several patches that have the same structure. This tool was intended to speed up the process of actually building the quilt block while maintaining the feel of using a manipulative.

Classroom Trial

Once a working prototype of a software version of DigiQuilt was complete, I tested it with students from two third grade classes at Some Elementary School. Some Elementary School reported that 56% of their 3rd grade students were eligible for free or reduced lunches that school year. The predominant races and ethnicities listed in the same report say that in the 3rd grade, 63% of the students were black, 22% were white, and 10% were Hispanic. In addition, the teacher told me that the school was located in a place where many students move in and out during the year, so the classes were often changing to welcome new students or say good-bye to others. Five students from one class and eleven students from another class signed up to participate in the study. As a part of this study, I gave the students a pretest and had them use paper pieces in an initial activity. As part of paper piecing activity, I told the students that they were going to be designing some quilt blocks that would be used as examples to help 2nd graders learn about fractions. The students were also told that some of them would be using the computer to create the designs, and some would use paper pieces. I also told them that I was interested in knowing more about what was hard about designing quilt blocks and understanding fractions, so they were welcome to ask lots of questions or discuss what they were doing with me or their classmates.

Results

The children set out to make paper or computer quilt blocks as directed. Since the last section focused on the use of paper pieces and covered most of the results from this study regarding the use of a system of paper pieces and grids, I will focus on the results of the software use by the children in this study for this section. The students had an easy time getting started with the software – picking up squares and triangles from the palette on the left and placing them in the block work-area. The learners using DigiQuilt were able to save and clear their quilt blocks. In fact, they cleared their quilt blocks more than I thought they would, seeming to prefer to start from a blank block work-area rather than making and saving incremental changes. They could view their quilt blocks as images in the folder on the computer where they were saved and print them on paper to share with their classmates or family members.

Although the challenge-driven approach worked just fine for keeping students engaged in designing quilts, now that two different design approaches were used, it was not easy to keep them all working on the same challenges. It was difficult to have the students share their quilt blocks between computer and paper groups. This difficulty was partly in that students were working at different paces and partly that the students working with paper often expressed interest in the software (and using it) more than in the designs their classmates were trying to share. Printing the designs was also not as easy as I had hoped. The designs were saved on the computers the children were using, and they could only be printed from outside of the DigiQuilt software. So, printing could either occur during the time allotted for DigiQuilt use (cutting into design time), or after DigiQuilt time was over for the day (which meant sharing designs from the computer screen rather than on paper).

I had hoped that users would be able to make their quilt block designs so much faster than with paper pieces that they would be able to learn more about fractions through the design process than those who used paper pieces. However, since the learners in the study only were able to use the software a few times, the learning curve of the interface, though fairly small, was enough to keep the students using paper pieces or DigiQuilt at about the same speed for making their designs. Also, since students tended to clear the block work-area rather than making incremental changes, they were not as able to take advantage of parts of their design that they could reuse.

In the process of looking at the software in use by third graders, I learned firsthand about some difficulties elementary-aged learners have when it comes to fractions. Challenges that involved equivalent fractions were difficult for students to understand. For example, when approaching the challenge, "create a quilt block that is 1/2 yellow, 1/4 red, and 2/8 blue," students ran into several difficulties. Some students did not understand that the three fractions added up to 1 and would result in the whole quilt block being filled. Some students began by filling the quilt block with 1/2 yellow, and then filled 1/4 of the remaining space with red. Since this didn't leave eight squares behind, some students would simply put a blue piece in the design to finish it. I thought that perhaps students needed to understand that each fraction referred to the whole quilt block. Using the paper version of the manipulatives in the classroom, it was easy to create and test a variety of methods for supporting students as they explored fractions and symmetry through their designs. In order to help the students refocus their attention on these different fractions, I drew heavy lines on a paper 16-patch base block: first just one heavy line to emphasize 1/2, then another line to emphasize 1/4, and finally enough lines to break the design into 8 equal parts so the students could finish the design with the 2/8 blue. Trying this idea out with paper was easy and the results were encouraging. This initial attempt to help learners focus their attention on the base block in a new way eventually led to the development of the “select-a-grid” tool, which will be described in more detail later.

Another difficulty was that some students were not ready to talk about the mathematics involved with their designs since the fractions involved were too far beyond their level of understanding. Having a 16-patch design made it easy to make fairly complicated designs. I thought that having the option to make designs with fewer patches would help alleviate this struggle so that students would not get too frustrated.

Adding Tools to DigiQuilt: Second Iteration

The classroom trial revealed several strengths of the design and uncovered several changes that might improve DigiQuilt and specify or extend the learning goals it could address (see Figure 10 for the version of DigiQuilt described in this section). At this phase in the software development, I guided the implementation work of fellow graduate student Jochen Rick and two undergraduates – Megan Chinburg and Marc Calahan. Chinburg implemented parts of the select-a-grid tool and a tool for swapping colors within a quilt block design. The color-swapping tool evolved to a structure-swapping tool implemented by Rick. Calahan implemented the tool for browsing through and opening previously constructed blocks. Rick added logging capabilities to support data collection about the software use. These three colleagues provided useful feedback on design ideas. Their help was crucial to the development of the software, and though I will continue to use the pronoun “I,” many people influenced and aided this work.

I kept much of the initial design and added several tools to the software after the initial classroom trial to both support learners' design process and math learning. In response to the results of the classroom trial that followed the first iteration on the design of DigiQuilt, I made several changes to the software, adding:

- a tool for selecting different grids,
- the option to use different base blocks,
- new pieces,
- new tools for changing patch-level structures that combined the old turning and copy snaps (to support making quick changes),
- the capability for students to view and open their designs after they are saved, and
- the ability for students to "undo" and "redo" changes to their designs (allowing them to navigate through the history of the design).

In addition, a "halo" effect was added to indicate the patch-holder that was being pointed at by the cursor. In the previous version, empty patch-holders would turn dark gray if they were under the cursor, but if there were pieces in the patch-holder, the highlight would not necessarily show.

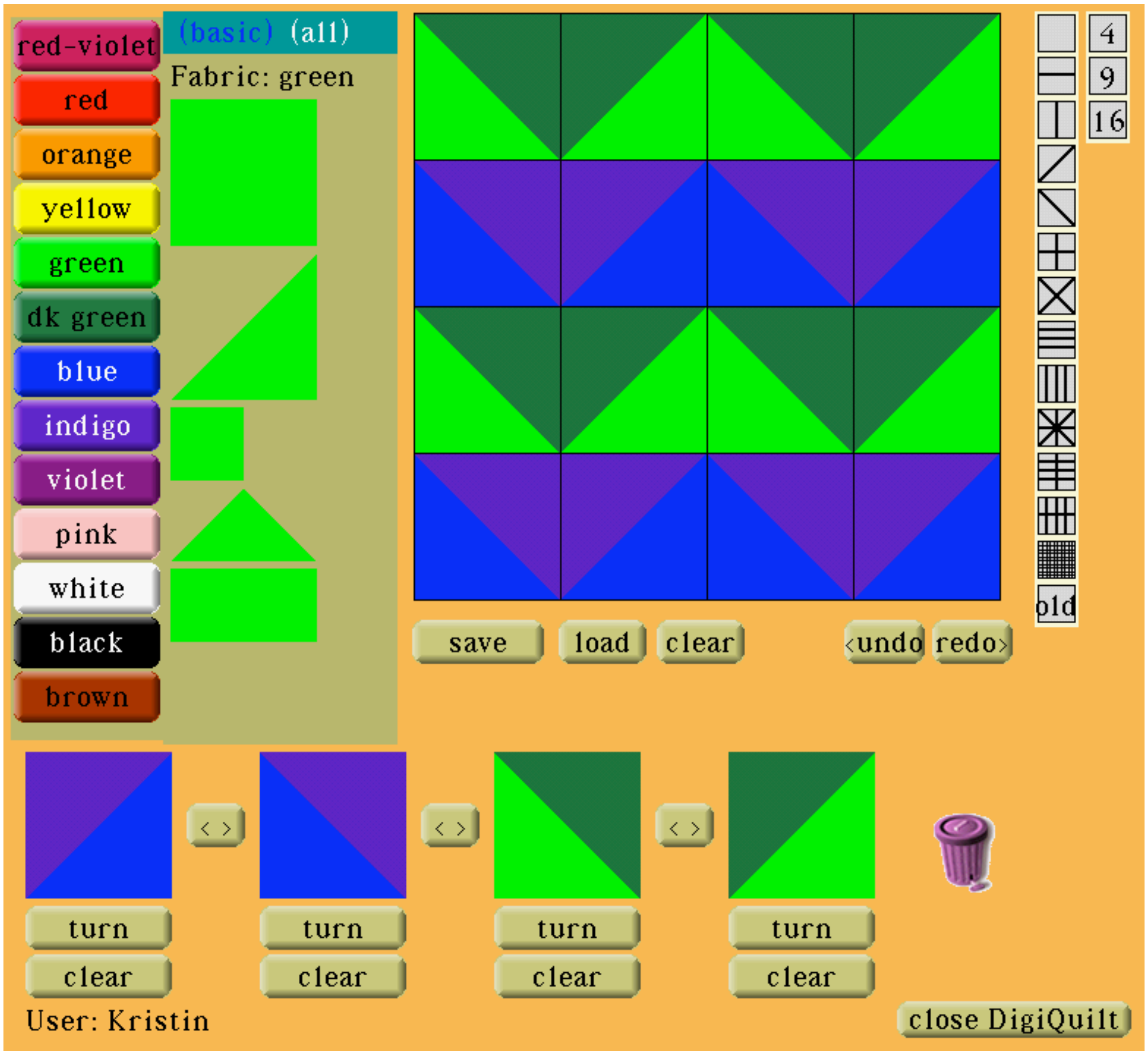


Figure 10. DigiQuilt with additional tools and options.

Design

New tool to support learners' understanding of equivalent fractions

One tool that came from this exploration was the "select-a-grid" tool (see Figure 10, second column of buttons from the right, and Figure 11 for examples). The select-a-grid tool allows users to impose a variety of grids on top of the quilt block design area. It adds support for student learning by highlighting connections between fractions in the challenges and the designs the students make. The main purpose of this tool was to help learners refocus their attention on the “whole” as they work on different aspects of the challenge so that they could understand how equivalent fractions can describe the same design; how different fraction-stories can be applied to the same situation. A secondary purpose of the select-a-grid tool was to provide grids with lines that can be lines of symmetry. The grids can serve as useful reference points for students as they attempt to solve challenges that involve symmetry.

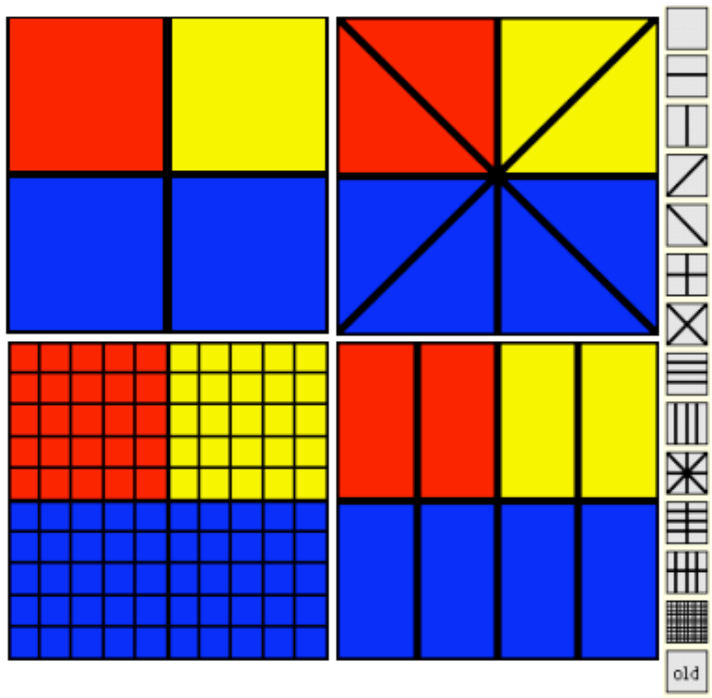


Figure 11. The select-a-grid tool allows learners to superimpose a grid on their designs. Shown here are four different grids superimposed on one design. To the right are the grid options available.

New pieces available

In the first version of DigiQuilt, students could either work in a mode where they used only squares, or they could use both squares and triangles. When I updated the software, I added some new pieces. Learners now had the option of using either squares and half-square triangles (which cover half a patch), or using those pieces plus a quarter-patch square, a quarter-patch triangle, and a half-patch rectangle.

With the 4-patch base block and the new, smaller pieces that are available, 1/64th is the smallest piece they would ever need to worry about, but it is much more likely that they would end up talking about fractions with denominators no larger than 16 because of the kinds of designs learners tend to make – you have to really go out of your way to get only 1/16th of a patch to be covered in a given color (see Figure 12). In the 16-patch base block, this tiny piece covers 1/256th of the total area, and in the 4-patch base block, it covers 1/64th of the total area. This size piece enables some interesting fractions.

Figure 12. A quilt patch with 1/16th of its area covered in white.

New options for smaller base blocks to offer a more mathematically constrained design space

In addition to the select-a-grid tool, I gave learners the ability to build their quilt blocks using 4-, 9-, or 16-patch base blocks in order to help students who were initially struggling with the math of their designs. In the initial trial with the 16-patch base blocks and the pieces software provided (which can overlap), the smallest piece the learner could create would cover 1/64th of the entire quilt block. With the new pieces that were added in this iteration, the designs could feasibly be much more intricate with quilt-patch-parts covering as little as 1/256th of a quilt block. Adding the option for a more constrained (smaller) base design reduced the complexity of fractions students could encounter. Since 4-patch and 9-patch quilt blocks are often found in traditional quilts, they fit with the overall theme of quilting and allowed learners to approach their challenges with just a little simplification. The addition of the 9-patch base block also allowed learners to make fractions like 1/3 and 1/9, which was previously impossible.

New facility for making quick changes

I also wanted to add the ability for users to change their designs easily without feeling the need to start from scratch. To accomplish this, I wanted to add a tool for users to change the colors of the patches in their quilt blocks. In this phase of design, I also realized that there was no particular need for the tools that rotated and copied shapes to be separate, so the two functionalities were combined into one part of a tool (shown in Figure 13). This set of specialized patch-holders is called the "patch work-area" since it is the work area for patch-level designs (as opposed to the "block work-area" where quilt blocks are constructed).

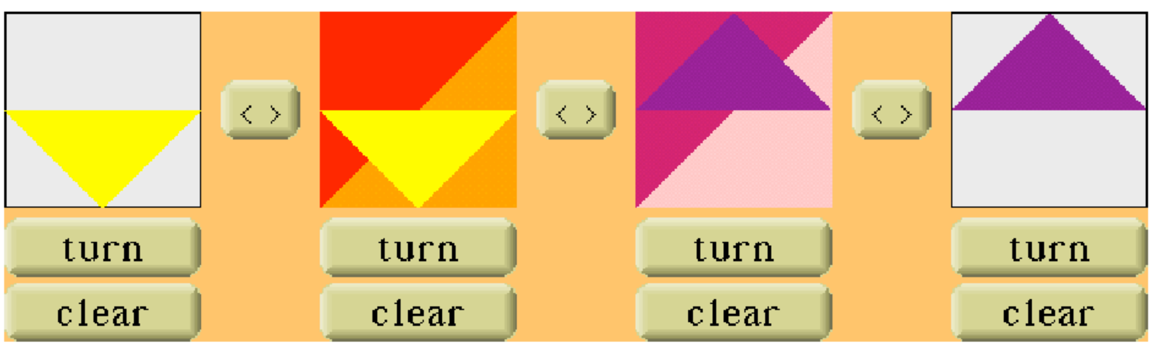


Figure 13. The patch work-area: a series of four patch-holders where patches can be constructed, turned, copied, or "swapped."

The idea to change colors (color-swapping) easily grew into the idea to swap patch-level structures instead (structure-swapping). Structure-swapping could be used for color-swapping, but also allowed for making larger revisions to a quilt-block design. The series of four patch-holders across the bottom of the screen has buttons between the snaps that allow the user to swap all instances within a quilt block design of one patch-holder's contents with instances of the other patch-holder's contents. This works for any rotation of the same patch-level pattern. I thought this would further enhance students' ability to learn about fractions through the design process because they can make changes and see the effects so quickly. Another (perhaps less intuitive) way this tool can be used is to leave one patch-holder empty and construct a patch in a neighboring holder. Using the "swap" functionality, all the empty patch-holders in the block work area will instantly be filled with the contents of the non-empty patch-holder (from the patch work area). This provides an easy way to fill a quilt block quickly.

New facility for students to view and open old designs

In order to make it easier to not only share designs, but also to start from a partially complete or complete design and make changes from there, a tool for users to view and load old designs was added (Figure 14). I thought that this capability, combined with the ability to swap patch-level patterns, would dramatically change the use of the DigiQuilt system. Students would be able to change their designs in interesting and significant ways with little effort. Students could also view and open designs made by their peers. Changes made to the designs would be attributed to the student who opened the design and would not change the design as it was previously saved. Learners would be able to compare many example designs just like with the paper-pieced quilt blocks, only now they could do so anytime and across classes rather than only during structured sharing sessions.

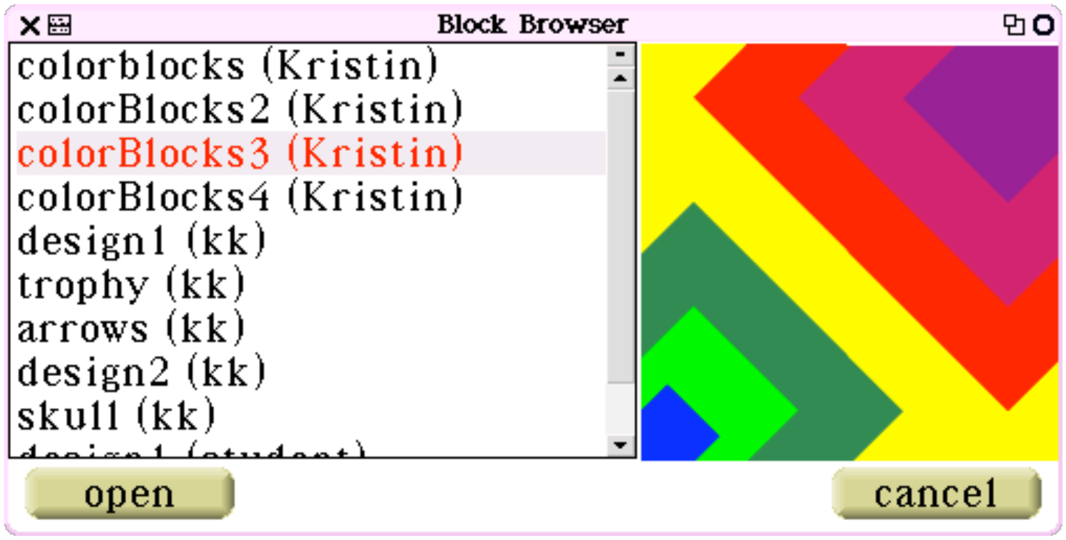


Figure 14. The block browser allows students to view previously created designs.

New facility for students to navigate through changes

Another important change to the software was the ability to navigate forwards and backwards through the changes made to a design. The ability to "undo" user actions allows the learners to experiment without worry about ruining their designs. Making it easy to recover from "mistakes" allows the learner more freedom to explore the system. Not only does this navigation feature allow users to try things and to undo them, it also allows them to step through the history of any design someone has saved in the system. This way, they can see how it is possible to make any of the designs others have made. With so many artifacts acting as working examples, I hoped users would be encouraged to try new ways of designing quilt blocks and daring ways of using the included tools.

Trial

Students in four classrooms from three different schools in two school systems used DigiQuilt software in the spring of 2003. This classroom trial was the first time I worked with such a large number of students. I enrolled 40 3rd and 4th grade students and 6 teachers in 2003. My goal was to test the usability of the system and to check whether affordances I thought were built into the system were recognizable by students. In the two schools within one school system, the whole class used the software at one time in a computer lab setting. In the other school, I brought laptop computers into the classroom and worked with about six students at one time. I did not request or obtain permission to videotape this classroom trial, and working with such a large number of students at one time made it very difficult to collect very much data (especially since when I was in the field I needed to orchestrate the activities as well as pay attention to what happened with the students using the software). As a result, most of the data from this trial is very informal. The purpose of this trial was to find out how students would use the newest version of the software, particularly looking to see how the students used the tools, what kinds of support the tools seemed to offer, and what kinds of support the students still seemed to need.

Results

I found that students were able to complete challenges with a mixture of fractions included, and that the select-a-grid tool helped them do that. However, the designs were also somewhat less interesting when the fractions were difficult (see Figure 15 for examples of designs students made using the select-a-grid tool). When students used the select-a-grid, they tended to fill in large blocks of color rather than creating intricate designs. This may stem from the fact that keeping track of how many pieces of each kind were being used (to figure out the less familiar fractions) was too hard without some kind of guidance.

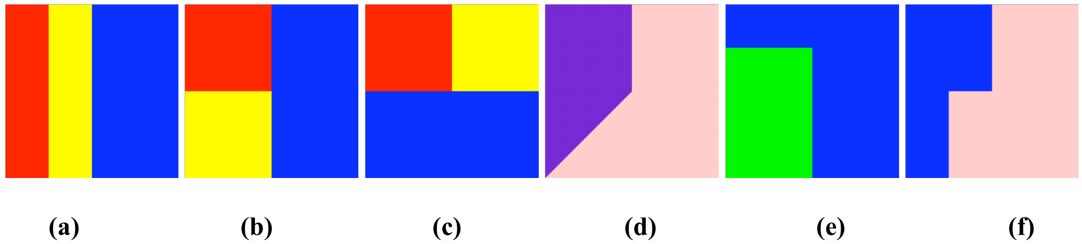
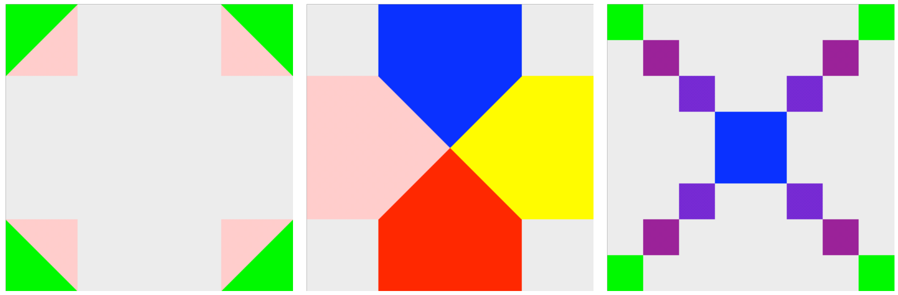


Figure 15. Designs from two challenges made by different students using the select-a-grid tool for support. (a, b, c), three designs made in response to the challenge to make a quilt block that shows 1/2, 1/4, and 2/8. (d, e, f), three designs made in response to the challenge to make a quilt block that shows 3/8 and 5/8.

In addition to fractions support, some learners used the select-a-grid tool for other kinds of design inspiration. The different grids seemed to lead students to look at and plan their designs in different ways both aesthetically and mathematically. In particular, the grids that divide the quilt blocks into triangles led several children to create designs with a central focus that seem to spin or begin in the center and work their way out like a spider's web (see Figure 16). However, some of these designs seemed to lose meaning for the children over time because the grids were not saved as part of the design (though perhaps that would be a useful option to consider for future versions of the software). One example of the designs losing meaning without the grids being attached is the Figure 16a. Its name is "spider's web" – a name that stemmed from the grid. Without the grid, the name lost meaning for the designer of the quilt block (later when she looked at it, she remarked that she did not know why she called it that). In addition, the name did not seem to make any sense to other people who could see the quilt block and its name, but could not see what the learner saw when the quilt block was named in the first place.



(a) **(b)** **(c)**

Figure 16. Designs made using the grids that separate the grid into triangular spaces.

The students mostly used the 16-patch base blocks. I think this was partly because it was the default style of quilt block, but also because the resulting designs could be so pleasingly intricate. The students seemed to be more impressed with complicated designs – they were more likely to share their designs when there was something unique about them, and the 16-patch base block left the most space for creativity. The 9-patch base block made it possible to work on challenges with certain fractions, like 1/3, that could not be successfully addressed using the 16-patch base block and the pieces provided.

Though I did make some smaller pieces available to students, thus adding some complexity, the learners began their interactions with DigiQuilt using only the big squares and triangles. This entry mode is labeled "basic" at the top of the patch palette. As I anticipated, learners in this mode rarely made fractions smaller than $1/16^{th}$. One reason that the tiniest fractions were rarely encountered is that many students end up making designs with repeating patterns that tend to have similar patches within one block. In general, designs with repeating patches are not only *simpler* (mathematically speaking), they are also designs that lend themselves well to helping learners see that the granularity of the pattern does not change the fractional coverage (a checkerboard with an even number of tiny squares or an even number of large squares will be covered half with one color and half with the other). Repeating patterns also enable more efficient use of the swapping tool in the patch work-area, though students did not take advantage of the swap tool very well.

The facility for making quick changes using tools in the patch work-area was not fully discovered by most students. As in the previous trial, they used the patch-holders to turn and copy pieces and patch-level designs, but they rarely used them for swapping. In the time available, most students seemed not to realize the full potential of the tools in the patch work-area.

The students were very excited about being able to use the software to look through the collection of designs they had made (previously, this could only be done outside of DigiQuilt). Tools to allow sharing their designs with other students were not fully developed by the time this trial happened, so students were limited to sharing their designs using the, “come here and look,” method. There was plenty of sharing that happened despite the limitations of the technology. In addition, some sharing via computer was possible since the computers they used were in the school computer labs, which are used by students in multiple grades, students were able to see designs made by children in other classes that were saved on the computer they were using.

Connecting Design Decisions to specific affordances suggested by the literature

At this point, the software seemed to be a viable tool for learning about fractions and symmetry through the design of patchwork quilt blocks. During the summer of 2003, I spent a good deal of time researching the math education literature to help me understand more about the difficulties that children have learning fractions and how the software might help children through them. I found some explicit support for design decisions I had already made, an idea about how to improve the software, and ideas for the software’s use in the classroom.

Support for design decisions already made

Most of the support for various design decisions has been presented along with the description of any added feature. One specific design decision supported by this literature review was the inclusion of a tool for helping the learners view their designs in different ways. Because these lenses are a particularly important part of the style of computational manipulative I believe I have created, I will highlight the tool in DigiQuilt that I consider to be a lens.

The "select-a-grid" tool allows users to superimpose various grids on any quilt block. It was developed based on my initial experiences with children using paper pieces. I needed to find a way to help children keep track of the whole as they were working with fractions. I tried to explain fractions as a story about a situation – a story that can be told from many perspectives in many different ways. Keeping track of the "of what" when dealing with fractions seemed like a good way to help children focus on understanding the fractions story with which they were working in a dynamic way. Kaput (1992, page 525) suggests that understanding invariance requires the presence of variation and that dynamic media make variation easier. The select-a-grid can be used to change the view on a fixed object (another idea mentioned in Kaput, 1992). Here, I think the tool works especially well because it highlights a lack of change. The design stays the same and the fractional area covered by a given color remains the same, but the grid changes, allowing learners to see that we can think about dividing the same design in a different way. Even though an action is taken by the learner, the relationship between the symbolic math and the concrete design is held. The tool helps the learner see the same design in a new way by changing the view of the designed artifact. The grid highlights both connections and expectation failures.

This literature search also revealed some of the benefits of computational manipulatives as a tool for math learning. Most of these benefits are detailed in the background chapter of this dissertation, but I will highlight again the benefit of having children use manipulatives in a constrained environment. Using the manipulative in a constrained environment limits the modes of interaction. Limiting the interactions allows children to use the manipulatives in a beneficial way (that allows them to connect concrete and abstract representations of mathematical concepts) without as much guidance as would be needed for the use of physical manipulatives. Depending on the manipulative and how closely it embodies the targeted concepts, these constraints can be very important for helping children connect the targeted concepts to concrete examples.

An idea for improving the software

The major change I made to the software based on this literature review was the addition of feedback about the fractional area of the quilt block that is covered by each color. This feedback is displayed as a reduced fraction on the button in the interface that corresponds to each color (see Figure 17 for a detailed view). This change was mainly due to research from my literature review that suggested that students would benefit from having the appropriate tools to monitor their own progress toward goals (e.g., Schunk, 1983, 1991, Schunk & Swartz, 1993, Pintrich & Schunk, 1996). Giving the learners the ability to check their progress toward goals addressed by the challenges also enabled them to do the same for their own personal fractions goals. The buttons are always displayed, so putting the fractions feedback there means it can always be displayed as well. This also affords discussing the math, as one can simply read it (e.g., The quilt block is ¼ yellow.)

Figure 17. A button showing the feedback for a quilt block that is ¼ yellow.

Ideas for using DigiQuilt in the classroom

Because I wanted to know how using DigiQuilt over time would help children learn about a variety of concepts, I knew I needed to find ways to help them get and stay engaged with mathematical concepts. I decided to try several strategies to promote collaboration and short-term engagement throughout the next trial, following several students' varying levels of engagement and motivation in conjunction with the implementation of these strategies. One way to help children engage more freely with mathematics, as suggested by Eisenberg (2003a), is to have them bring mathematically inspired artifacts into the world. This inspired the idea to allow children to share their quilt designs using stickers or cards that they can trade since collecting things can be a big draw for children. I decided that throughout the dissertation study I would introduce new ways of sharing quilt block designs in order to understand how these options for sharing impacted children's conceptions of their audience and their design habits. I anticipated that this sharing would increase children's motivation to create interesting designs that were meaningful to them and that they thought others would enjoy. I hoped that children would surround themselves with beautiful, mathematical creations. The strategies I planned to implement to try to keep children engaged and support collaboration included new ways of sharing quilt block designs (printing on magnets, iron-on transfers, and business cards), new tools for designing and understanding designs (tools like the select-a-grid tool but with different foci), and new opportunities for participation (sewing real quilts, critiquing designs, or playing games that involve the quilt designs).

A brief description of DigiQuilt as it was for my dissertation study

In the end, DigiQuilt's design supports both personal and epistemological connections for most of its users. DigiQuilt has features to support math learning and design, and some features that support both in a variety of ways. I thought this freedom would allow some learners to have the coveted "gears-like" experience I was after, while the supports for bridging between the abstract and concrete would engage all learners on some level. I will describe the affordances of DigiQuilt as it was for my dissertation study in terms of its affordances for design (mostly personal connections) and math learning (mostly epistemological connections).

Affordances for design

The design-oriented nature of the software is meant to allow learners to create designs that are meaningful to them. Some of the tools in the software are explicitly designed to support the design process – DigiQuilt has several facilities that make quilt block composition easy. One tool allows users to copy a group of pieces that make up a patch to repeat the design in multiple patch-holders. Using this feature saves time when designs use repeating elements. The hypothesis driving the decision to include this facility was that learners using DigiQuilt will be able to create comparable designs more quickly than with physical manipulatives and that they will be able to complete more complex designs in a given time than they could using physical manipulatives. I imagined that the ability to complete more complex designs or to make more designs would afford deeper learning of the material, either through stepping through a more complex design process or through repeated creation of simple designs that focus on the variety of possible solutions for a single challenge (e.g., creating several representations of one-half that are symmetric from side to side). This multi-purpose tool that allows users to turn or copy pieces and patches can also be used for swapping two patch-level designs wherever they occur in the block work area (regardless of the rotation of the same resulting coverage). Since many quilt block designs utilize repeated patch-level patterns, the ability to swap patch-level patterns throughout the design with a single press of a button allows users to quickly explore rather complex changes in their designs without very much work.

Affordances for math learning

Other tools support "seeing" the math. A tool for selecting a base block (framework) for the quilt block in the block work area offers users choices between 4-, 9-, and 16-patch designs. This feature was added because I noticed that some students had a difficult time understanding the fractions they were creating as they designed quilt blocks in the original 16-patch-only design. Since I wanted the objects the students created to be catalysts for mathematical discussions, I added this tool to make the math more accessible without taking away the framework for more complicated designs.

To help learners connect the concrete representations they manipulate in the software to the symbolic representation of a fraction, DigiQuilt constantly updates the symbolic fraction on the color-buttons so that it matches the spatial ratio of the quilt block in the block work-area. The feedback about fractions is designed to help children explore fractions more deeply than they might be able to without the feedback. The feedback allows them to set their own goals about cool fractions they want to try to create and work on their own, or to check their progress while solving a challenge. This affordance most clearly supports math learning, but it also supports design efforts by not requiring that quilt blocks look a certain way – the pieces of a certain "fabric" can be in any arrangement and the fractions feedback still works. By not forcing the learner to step out of the design realm to check on the fractions, the feedback supports children's fractions explorations within the context of their design activities.

The "select-a-grid" tool (see Figure 11) allows users to choose a grouping of lines to overlay on the quilt block in their workspace. The grids divide the quilt block into 2, 4, 8, 16, or 100 equally sized pieces in several different ways. These grids are meant to be used interchangeably on a given design to highlight several mathematical ideas and offer students structure as they approach their designs. They show that designs can be broken into pieces of different sizes without changing the fractional area covered by a given color. This is one way for learners to notice how equivalent fractions could possibly be equivalent. They can be used as supports to build parts of the design and changed to complete the design without disturbing the layout of the pieces. Changing between grids can help the students refocus and change perspective, at the same time noticing that their design does not have to change just because the way it is divided up changed.

The select-a-grid tool serves several purposes for learners – one of which is to help them with the learning goal of understanding equivalent fractions by connecting one design with several ways of partitioning the space and thinking about the spatial ratio. Preliminary findings suggested that helping students tell a variety of stories about the same design was a useful step towards understanding what a fraction can tell us about something or how to interpret the meaning of a fraction. I also noticed that learners often needed help understanding the idea that rearranging parts of the whole does not alter the portion of the whole that the parts comprise. The "select-a-grid" tool is meant to support explorations that help learners notice this. When a learner chooses a new grid, the design stays the same, but the grid helps them see the design in a mathematically new way by helping them attend to it differently.

Plan for using DigiQuilt in the classroom

Learners using DigiQuilt in the classroom complete challenges that ask them to focus on fractions and symmetry, e.g., "Make a quilt block that is 1/2 one color and 2/4 some other color," or, "Make a quilt block that shows 5/16 and has one line of symmetry." The challenges are developed with specific learning goals in mind. In the first example, the goal is to introduce equivalent fractions; a concept that is particularly difficult for many children to understand. The second example can be challenging because the learner must choose and place pieces carefully to meet both parts of the challenge. The feedback the software provides about fractions does not limit the students' design activities, but provides a way for students to monitor progress toward their personal goals or the goals set forth in challenges provided by the teacher. In addition, the feedback combined with the challenges sets up many opportunities for expectation failure. DigiQuilt has similar affordances to other computer-based manipulatives and microworlds in that it continuously updates the relationships between representations, allowing the learner to focus on the effect their actions have on the math of the design (Clements, D. H. & Battista, 2000), but it goes beyond helping students build bridges between the concrete and abstract by integrating opportunities for creating personally meaningful designs.

CHAPTER 4

RESEARCH CONTEXT, PROCEDURE, AND PRELIMINARY DATA THAT GUIDED THE STUDY DESIGN

Research context

In gathering data for my dissertation study, I worked with 4 classes in 2 schools within one school system near Atlanta. From my perspective as a visitor, both schools were great places for students to learn and felt very welcoming. However, considering that they were in the same system, they were quite different. Some of these differences were important because they impacted the types and numbers of activities these children engaged with outside of school. Since a major goal of this research was to create a tool children would be interested in using or adopting as part of their lives outside of school, I will describe some of these differences before describing each of the four classroom settings.

Statistically, the main differences between the two schools were their racial and socioeconomic profiles. According to the Georgia Department of Education annual report cards for the two schools, the majority of the students at CH elementary were black, while at GW Elementary the majority of the students were white (though the racial profile was more balanced at GW). At CH Elementary, 64% of the students were eligible for free or reduced lunches, while at GW Elementary, only 35% were eligible. Those school-wide averages are interesting on another level – the classroom and grade-level percentages at CH Elementary suggest that the younger students in the school had lower percentages of students with a low SES. 88% of the students in the 5th grade at CH were disadvantaged, while only 30% of their age peers at GW Elementary were disadvantaged. These differences seem even greater when you consider that there were only 1.6 miles between the schools. Though students at both schools had access to fairly equal levels of technology in the computer labs, from a strictly financial perspective, CH students were clearly less likely to have a computer at home.

Table 1. Socio-economic and racial statistics provided by GDOE.

	Disadvantaged	Not-disadvantaged	% Disadvantaged	Students with Disabilities	Asian	Black	Hispanic	White	Multi-racial	Special Ed. (school)	Gifted (school)
CH4	12	4	75%	1	1	15	0	0	0	19.8%	4.2%
CH5	14	2	88%	0	0	14	0	1	1		
GW4	12	11	52%	3	0	11	1	11	0	15.0%	11.6%
GW5	7	16	30%	6	1	7	0	13	2		

At the school level, there were several other differences that might have had an impact on the overall feel of the schools. In addition to the statistics mentioned above, the Georgia Department of Education reported rates for both giftedness and needing special education that widened the gap between the "haves" and the "have-nots." At CH 4.2% of the students were identified as gifted, while at GW it was 11.6% of the students. On the other end of exceptionality, CH had 19.8% of its students in special education, while at GW 15.0% were identified as needing special education.

These statistics show a certain level of financial disparity, but alone they do not illustrate the uniqueness of both schools and of each classroom as part of my study. In addition to the statistics that make up the profiles of the schools, there were some logistical differences between the schools that affected my research. In GW Elementary, students did not change classes for math, while in CH the 4th and 5th grade students changed rooms for math and language arts (the 5th grade teacher taught math to both groups of students). For me, this meant that I worked with 3 teachers as opposed to 4 or 2. I had less flexibility at CH elementary with the 4th graders since I only had access to them during the first half of their scheduled time for math (before lunch). As far as computing resources, at CH Elementary, each student was assigned a laptop computer to use for the year. In March, the students were able to take their computers home for the first time. At GW, the students used the software on laptop computers from a cart. The students generally used the same computer from week to week, but the computers remained at the school. It was easier to access computers at GW from my perspective since I could predict and plan for when the computers would be available. Data collection was easier since it did not rely on finding time between other student activities.

In the next few paragraphs, I will paint a more descriptive image of each class, how these classes differed from each other, and what unique perspectives or opportunities they offered for my dissertation study.

CH Elementary Description

Mr. S, the 5th grade teacher (who also taught math and science to the 4th graders at CH Elementary) was enthusiastic to have the students participate in the study. His teaching style suggested that he valued teaching the students to be independent thinkers. Students were held accountable for their learning and understanding, and he seemed to encourage them to think for themselves. In fact, at CH Elementary, after the Pledge of Allegiance, all the students in the school participated in another pledge of sorts – one that emphasized individuality and self-worth. Overall, these students began the school day on a positive note. Most students in these classes ate the school-provided lunch (not surprising since

88% of the 5th grade students and 75% of the 4th grade students were eligible for free or reduced lunch).

There was no recess at CH Elementary, but Mr. S often took his pupils outdoors to let them run, play, and generally work out some energy. One activity that seemed important at CH Elementary was that throughout the school year, the 5th graders prepare for their big presentation on Black History Month, which includes a performance for the school community including families and students. Another activity that mattered to the students was "bank day" – once a week the students were encouraged to bring in some money to put in the bank. This activity encouraged children to learn about planning and saving their money for larger goals.

The participation rates in my study by both 4th and 5th grade students at CH Elementary were high. There were 16 students in the 5th grade at CH Elementary. 14 students brought back permission forms, and all the students who brought permission forms also chose to assent to participate in the study. 7 of the 14 students had used the software for a short time in the spring of the previous school year. In the 4th grade, there were 16 students according to GDOE, but by the time my study began, there were actually only 13. Of those students, 11 brought back permission forms to participate in the study. 6 of the students had used the software in the spring of the previous year. So, at CH, 13 of the participants had experienced DigiQuilt at least one time, and 12 had not.

One unique thing about CH was that each of the students in 4th and 5th grade had a laptop computer assigned to them as the result of a grant. They used their laptops throughout the day in many subjects. For my study, that was exciting news because it meant that children might have the opportunity to use the software outside of class if they wished. When I was planning my study, I intended to find out as much as possible about how students used the software at home. In the end, the students were not able to take their laptops home until March. That alone changed the study quite a lot, but even more surprising was that after a few weeks bringing home their computers, the students seemed to mostly stop bringing them home at all. When I asked one student why she did not bring home her computer, she told me it was because she did not have anyplace to plug it in to charge it. Other students mentioned not wanting to carry their computers to school. One child mentioned that her grandmother would not let her use the computer if she did not share it with her sibling – the same sibling who had previously broken off keys and was under the age of 3. A culture of bringing home the laptop computers just never developed in these classrooms to the extent I had thought and hoped it would.

GW Elementary 5th Grade Description

Mrs. H's class at GW Elementary included 23 students, many of whom had gone to school together since kindergarten or first grade. They would be the last class at GM elementary to complete K-5 under one roof since the school system was restructuring the schools to uphold desegregation laws and GW would soon house only 4th and 5th graders – all the 4th and 5th graders in the small neighborhood-school system. Mrs. H had most of these students in class for both 4th and 5th grade, and at GW, students did not change classrooms or teachers for any subject except art.

Mrs. H had the tradition of sewing a quilt with her 4th grade students after completing a unit on the history of quilting in her classroom. Thus, these students were especially knowledgeable about quilting (both when they participated in the spring of 2003, which was during their unit on quilting and before they sewed a quilt as a class, and in this study). Their experiences using DigiQuilt must have been different than if they had no prior knowledge about quilting.

An activity unique to GW was the concession stand run by the 5th graders as a fundraiser for their class trip. Each Friday throughout the school year, the 5th graders and several 5th grade class parents would run a concession stand where all the students were allowed to buy treats. They used the money they raised for an overnight trip. The 5th graders could volunteer to help. When I worked with these students (on Friday), helping at the concession stand was one of the competing activities during the free time that followed DigiQuilt time. In addition to concession stand, the Friday afternoon free time at the end of the day was students' only time for several other fun activities. On more than one occasion, the privilege was either revoked or replaced by another whole-class activity, so DigiQuilt was not an option since there were *no* options.

From Mrs. H's class, 18 out of 23 students brought back permission forms. All of the students chose to participate and signed assent forms. 11 students used DigiQuilt the previous year as part of my study. This was the only group of students in my study where more than half of the students were not considered economically disadvantaged. However, the main reason this class was of particular interest for my study was their previous experience with quilting.

GW Elementary 4th Grade Description

The 4th grade class at GW Elementary was almost entirely made up of students who had never used DigiQuilt (the one exception was a student who was not promoted to 5th grade with the rest of Mrs. H's class). The 4th grade class at GW Elementary was the most balanced from the perspectives of socio-economic status (12 disadvantaged and 11 not-disadvantaged) and race or ethnicity (11 black, 11 white, 1 Hispanic). Overall, I found these students to be the most agreeable – they took direction well, listened to me and to their teacher, and were generally enthusiastic. This might have been simply a matter of perception, but their teacher also seemed to have less need for getting upset about their behavior, so I do not think it was completely a matter of perception.

From a logistical standpoint, there were several reasons why collecting data from the GW 4th graders was easier than any other classroom. First, I was the least rushed when working in this classroom because I had their recess time to set up the computers (and 2-3 helpers each week who helped me get set up). Second, they were the only class I met with on Thursdays, so I did not have to hurry to get anywhere else. Finally, this was the only class where I knew I could let their designs sit for a day before needing to worry about moving data around. Due to limitations on both the software and the network connections at these schools, I was unable to automate the file sharing of the quilt block designs that the children made. However, since I felt it was an important aspect of their experience to be able to see designs made by other children, I would transfer the files from each computer to the server and then download the complete set to each computer. This process was time-consuming, so I did not "harvest" data from the computers until after their 5th grade counterparts finished their work the following day (each week).

19 students out of 23 brought back permission forms to participate in the study. One oddity in this class was that the students who did not participate (those who did not bring back forms at all) were all eligible for special assistance due to poor math performance. During our DigiQuilt time, then, they went to a math resource room to get some more individualized assistance. From a research perspective, it was easier to collect data since the students who were not participating in the study were not present – I did not need to avoid videotaping them or sort out data that I could use.

Procedure

For each of the four classrooms, I collected a lot of similar types of data, but I did so in slightly different ways for a variety of reasons. The data collection procedure can be broken into three phases for each class: pre-DigiQuilt-use data, DigiQuilt-use data, and post-DigiQuilt-use data.

Pre-DigiQuilt-Use Data

Before the study began, I knew I wanted to collect data that would tell me about children's fractions knowledge and the ways they used manipulatives to help them solve problems. I also knew that I wanted to know something about the kinds of things children participated in outside of school in their free time, or their interests and hobbies. I knew that I would have limited assistance in observing students in the classroom and that I would need to be able to reference video data in order to keep track of what happened in the field.

Permission from parents allowed for separate permission for participating in the study and for allowing video data to be used in presentations (all students would possibly be videotaped as part of participating in the study). That way, I knew I could videotape all participants to collect data for my study, and that I would just need to be careful not to videotape children very much if their data could not be used in presentations. Only one student's video data was restricted in this manner, so I simply avoided having this student captured by my stationary cameras on any given DigiQuilt day.

I used the pre-DigiQuilt-use data to help me choose my focus students from each classroom – in each case choosing students to videotape who were considered exceptional in art or math. I also tried to choose students who seemed like they would be willing to talk with their peers and me about their experiences. I wanted to choose students who were seen as good at math or art by their peers.

Interviews before DigiQuilt use in the classroom began

Before the students used the software as part of my dissertation study, I interviewed them individually to find out their interests and uncover some nuances of their understanding of fractions and their abilities to utilize manipulative aids without much support. These interviews were either video or audio recorded (the earliest interviews were audio-recorded since that was what the permission forms indicated for that first set of students).

To find out their previous knowledge about fractions, I asked some questions about fractions that would uncover their notions about the relative size of symbolic fractions. I asked several questions (developed by researchers who were part of the Rational Number Project (RNP) about fractions concepts. I was mostly interested in students' understanding of equivalent fractions and their ability to compare fractions and find the larger fraction (RNP, 2003). I wanted to see them work through some problems to see what strategies they used and how they talked about fractions. I was interested in seeing them use manipulative aids to solve problems. I provided physical manipulatives to allow me to observe some of their problem solving strategies for using various manipulatives. I wanted to see how readily they connected symbolic fractions to the concrete representations they came up with using the manipulatives.

Because I was initially interested in understanding as much as possible about how and why children's engagement would wax and wane over time, as well as whether or not they were interested in using DigiQuilt outside of school at all, I needed to know more about their activities outside of school. Did they like making things? Did they consider themselves to be good at math? Good at art? What kinds of things did they do in their free time? These are the kinds of questions I asked in the pre-DigiQuilt use interviews in order to find out more about students' interests.

Paper and Pencil Pretest of fractions knowledge

Once all the students had been interviewed, the students took a pre-test to measure their understanding of fractions. Since symmetry was not as central a concept in my mind for the students to learn (it was meant to be more of a vehicle through which they could explore some challenging design strategies), I did not ask any questions about symmetry in the interviews or the pre-test. The students did not have access to manipulatives for the pretest, though they were told they were allowed to draw pictures if they wished, and many of the questions asked them to explain their reasoning.

Once the preliminary data was collected, the students were able to begin using DigiQuilt. Since the process in each classroom was slightly different, I will present each of the 3 different situations or conditions under which the software was used.

After the students were done using DigiQuilt in their classrooms for several weeks or months, each student took a post-test to measure their understanding of fractions and symmetry. Finally, at the end of the study I interviewed several students from each class to capture on video their descriptions of several designs and have them solve fractions problems in a semi-structured interview setting.

DigiQuilt Use

After the preliminary data was collected, each classroom in turn began using DigiQuilt. My procedure varied by teacher to varying degrees, and each classroom of students had slightly different experiences. The following subsections explain the procedures and data collected for each classroom. For each classroom, I collected: some video data of the children using the software (though details varied), log file data that tracked important actions in the software, and designs the children created using DigiQuilt. I also collected some information about designs children requested in special formats like business cards, magnets, or iron-ons, challenge sheets with design names and other information written on them, and some information about technical difficulties experienced by participants.

A timeline

I ran studies in each of these classrooms semi-simultaneously over the course of 6 months. The starts of the studies were staggered with the CH 5th graders beginning in December 2003. CH 4th graders and GW 5th graders began in February, and GW 4th graders began participating in April (see Table 2). The lessons learned in the first three classrooms helped me refine my data collection and classroom procedures. I will describe the procedure and results for both grades at CH Elementary together since they were mostly the same and took place in the same classroom. I will describe the procedure for 4th and 5th grade at GW Elementary separately since there were two different teachers and the procedure differed as a result of lessons learned form the earlier parts of the study.

Table 2. Chart showing dates various classrooms used DigiQuilt software. All GW4 dates were actually the day before the date listed on the table.

	12/5/03	12/12/03	1/12/04	1/16/04	1/23/04	2/13/04	2/17/04	2/20/04	2/27/04	3/5/04	3/19/04	3/26/04	4/2/04	4/16/04	4/23/04	4/30/04	5/7/04	5/14/04	5/21/04
CH5																			
CH4																			
GW5																			
GW4																			

CH Elementary Procedure

At CH Elementary, use of DigiQuilt was completely optional. On Fridays, Mr. S had his classes play math games rather than holding class as usual. The games were generally played by groups of students at their desks, which were arranged in groups. There were usually enough adults in the room to guide 2-3 groups in games while the remaining group played games against east other without an adult. DigiQuilt was one of the options during these Friday game times. The DigiQuilt activity was guided by challenges I provided either one at a time on note-cards or all at once on a “challenge sheet” where students could record the names of their solutions.

The students worked individually on a laptop computer, but near other students. My goal was to have 4 students using DigiQuilt at the same time, though sometimes as many as 8 or as few as 2 were involved. I generally set up the computers so that 2 students would be seated next to each other and facing the other 2 students. Cameras were pointed at the screens of two students and could often capture the expressions and off-screen activities of the two children across the table (see Figure 18). Since the students seemed interested in talking to the cameras, I told them they could feel free to tell the cameras anything they wanted to about their designs (“camera talk” is described in detail in Lamberty & Kolodner, 2005). Additionally, students were encouraged to discuss their design activities or the math challenges with their peers.

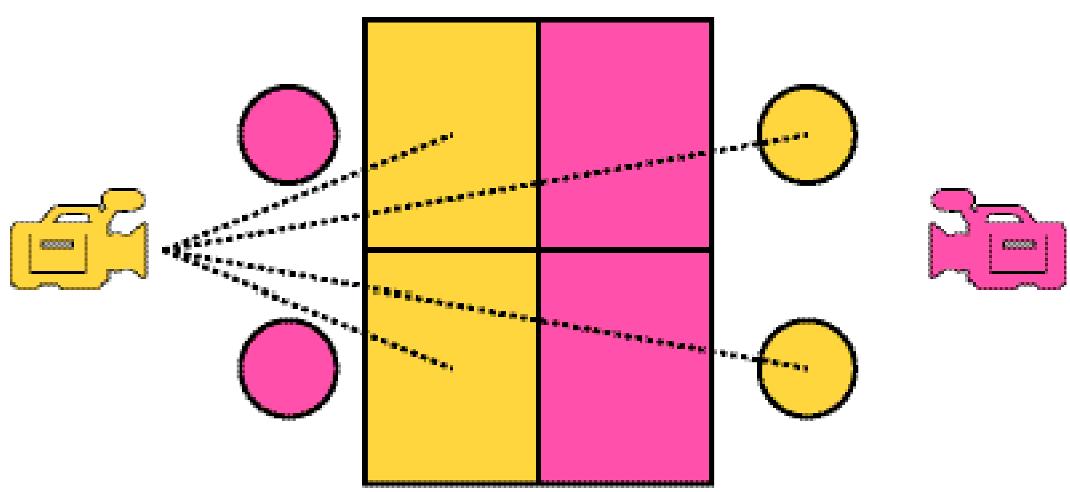


Figure 18. Camera set-up to encourage camera-talk and record DigiQuilt use.

On any given Friday, I would attempt to let students work for about 15 minutes at a time on DigiQuilt before changing to another activity. Sometimes students would choose to stay longer, or other students would decide they did not want to work with DigiQuilt when the opportunity arose, which might lead to a backup (too many students and not enough computers) at one moment and empty computers the next. One of the activities that competed for students' interest the most was the participation or watching of tournaments of various math games organized by Mr. S. Sometimes, I set up the computers in the back of the classroom, while other times I set them up in a resource room down the hall. If we were in the classroom, another game or activity would sometimes distract students. In the resource room, students might stray in the hallway walking between their classroom and the resource room. These may seem like insignificant details, but recognizing all the many things happening in a real school setting rather than a laboratory is important – and it varied by day, school, and student how these details impacted the students' participation as well as my perception of it.

The students were allowed to take home their laptop computers beginning in March, but only a few students continued to take them home after the first 2 weeks. Leaving the computers in the classroom rather than taking them home was disappointing for me in the context of this study since I hoped to find out how they would use the software in their free time, but since I was not there every day and it was not the main focus of this study, it is outside the scope of this work to discuss it in much detail. Collecting data from these computers was especially difficult since I did not know in advance when the students would need their computers for other classes (especially language arts). In addition, putting the students in charge of their computers led to at least one student not having a working computer to take home for at least 2 months before I was aware of the problem. She had been told her computer was "in the shop", so at school she was using an "extra" computer and she never took the computer home. Another student reported not taking her computer home because she had no place she could plug it in to charge it. Keeping track of the status of the laptop computers was difficult. With an already overworked technical support crew, CH Elementary students' laptop computers sometimes fell into disrepair and off the radar of teachers.

GW Elementary 5th Grade Procedure

At GW Elementary, the 5th grade students used DigiQuilt as a whole-class activity for about 30 minutes one day each week (except when scheduling conflicts arose). Because it was part of a whole-class activity, all the students in this class used the DigiQuilt software and I simply did not videotape students who were not part of the study. The activities were challenge-driven at this school as well, but since there were so many students using the software at one time, I relied on challenge sheets rather than note-cards. The GW 5th graders sometimes used the software without a challenge sheet for part of the time while we were getting set up for the day (and seemed less likely to stay with the challenges once they had a sheet than their 4th grade counterparts). Since there were so few cameras and so many students, "camera talk" was not encouraged as much and did not become part of the students' routine.

Most of the data collected from this class was video data. I videotaped 1-2 "target" students (who had been identified by their peers in interviews as being exceptionally good at math or art) and a group of several other students (selected more randomly) each day. I sometimes found a way to use a roaming camera in addition to the cameras on tripods, but some days were just too busy for that additional data collection. I collected challenge sheets as well, but many sheets were either blank or incomplete. I had much less contact time with the students at GW than CH since the whole class used the computers at the same time.

The GW 5th grade students all used DigiQuilt for about 30 minutes and then they were free to do whatever they wanted for the last 30 minutes of their school week. Since that free time only came once per week on Friday (and there were many activities that were options only during that time), very few students would continue using DigiQuilt into their free time on any given Friday. At GW, about 10-12 students brought permission forms and were able to take home the software to use on their home computers. I ended up not collecting any data from students' home computers directly, so the only data I have about their home use is whatever they mentioned in class or post-interviews.

GW Elementary 4th Grade Procedure

The GW 4th grade class was the last one I added to the study. In this 4th grade classroom, the students who did not bring back permission forms turned out to be 4 students who needed extra math help, so they worked with a math resource teacher and were not in the classroom during the time we spent using DigiQuilt. Because the other three classrooms in essence, helped me understand better how to manage the socio-technical system (the integration of the software into the environment and how to manage and facilitate its use) and how best to collect my data, the 4th graders at GW Elementary had, perhaps, the most tightly organized DigiQuilt experience. My time with them was more constrained as a result of our late start, but by the time I started this last study, I knew more about how I wanted to do things (which challenges to use when). I arranged to visit the GW 4th grade classroom on Thursdays, so that it would be the only class I visited those days. That way, I had the most time to get set up (while they were at recess) and was less rushed to get from one place to another. In this class, the exact amount of time available for DigiQuilt use varied, but was generally about 45 minutes. The students did not have other options, nor did they have any other time in the classroom for using DigiQuilt. I rearranged students' seating arrangement occasionally so that I could videotape students who had been identified as being exceptionally strong at math or art, but at the same time make it look to the students like I was changing it around more than I really was. The students seemed quite aware of who was being videotaped from one session to the next, so I tried to move things around and videotape people with whom my targeted pupils interacted. I also videotaped one student who I knew would talk to others more readily and who was not known for being good at math since I wanted to capture the interactions that might lead a student to a better understanding.

Post-tests and other post-DigiQuilt-use data

After the students were done using DigiQuilt in the classroom, I attempted to collect post-tests from all students and conduct post-use interviews with several students. I wanted to collect this data so that I could track changes in students' understandings of fractions concepts. I also wanted to see if they could do symmetry problems even though those were not on the pretest. I was particularly interested in looking at children's understandings of equivalent fractions and their ability to compare the relative values of fractions.

In my enthusiasm for having students use the software as much as possible, I left this part of the study until too late in the academic year. As a result, I was not able to collect as much data after classroom use of DigiQuilt ceased as I had hoped.

How the data collection procedure related to my goals and analysis

At both schools, in all 4 classes, my original intent was to track student engagement with DigiQuilt. I wanted to see how students engaged with the software, and the effects that different aspects of the intervention – tools, activities, and options for sharing the quilts – would have on the engagement of students. Over time, I realized another interesting story unfolding before me was actually about the different learning opportunities for students in the context of using the software. While engagement is important, the kinds of things that are happening while the students are engaged can inform the design of similar socio-technical systems. In particular, I wanted to understand what aspects of the socio-technical system seemed to help children connect abstract ideas to concrete examples, and how the children would integrate their personal experiences into their designs (or integrate their designs into their personal experiences). What happens when learners equipped with tools for helping them make connections between their (in-progress and completed) artifacts and targeted concepts participate in a challenge-driven design experience?

By the end of this study, I had collected an overwhelmingly large amount of data from student participants in these 4 classrooms, with nearly 1500 quilt designs and many hours of video data. I had pretest and interview data from prior to students' use of DigiQuilt. I had video, log file, and design data from their time using DigiQuilt. I had post-test and some post-interviews from after their DigiQuilt use.

I had originally planned to track students' engagement closely, both in the moment and over time. I learned in my pilot studies that the student participants would be very cognizant of who was being videotaped on any given day, and I wanted to be sure to capture a variety of experiences. Choosing more focus students from each class than I had cameras meant that I could not I could not follow students' individual progress closely enough to glean nuanced reasons for their increased or decreased engagement from day to day. The story that emerged more clearly from this study seemed to be about *how* children used the design environment to create quilt blocks that they cared about and that were the focus of mathematical discussions. These stories seemed to be the kind that happened in terms of shorter moments or event rather than as part of a more detailed history of a single student's engagement over time. However, early attempts to describe the overall story in terms of these shorter moments were unsuccessful. Indeed, telling the overall story for at least one class seemed necessary in order to give an overall feel of what it was like for the students to use DigiQuilt in a more contextualized sense. I needed to choose one dataset to help me lay out the possibilities.

Learning from the first 3 studies, refining the study for my focus class

I had some difficulties with my data collection process that I did not anticipate. Students at CH who had problems with their computers had a hard time reporting what was wrong in more detail than, “my computer isn’t working.” This sometimes resulted in students not having access to their laptops outside of school or using one of the spare laptops for extended periods of time, but being unable to take the computers home (even after that was finally allowed in March). “Harvesting” data from laptop computers that students needed to use for other classes (and then might possibly take home) was difficult and often didn’t happen on the weekly schedule I had planned. There was not a good time to collect the data from these computers other than while the students were using DigiQuilt. Some of the technical and logistical difficulties I encountered at CH Elementary resulted in delays in collecting data or incomplete data from the 4th and 5th graders at that school. In GW 5th grade, the students used DigiQuilt as a whole class activity, but not all the students were participants in the study, so collecting video data was difficult.

Because it was the last class I added to the study, and logistical reasons made the data collection process was the simplest, the dataset collected form the GW 4th graders was the cleanest and most structured dataset. Though all of the classrooms gave me interesting data, the data from the other classrooms works better as supporting data rather than guiding data. I used the experience and data I was gathering in the first 3 studies to inform my approach for the 4th study. Because I wanted to use my data to describe the *possibilities* when learners are equipped with tools for helping them make connections between artifacts and targeted concepts and challenges to steer them toward interesting activities, I focused on the data from the GW 4th graders for laying out that range of possibilities.

The next chapter presents a day-by-day account of DigiQuilt use in the GW4 classroom. I will focus on this particular classroom for several reasons:

1. Because they were the last classroom of students to join the study, these students' experiences were the closest to what I would term ideal
2. This classroom had the best cross-section of students participating – the students come from a variety of socio-economic and racial backgrounds and was split nearly equally between boys and girls
3. Although there were only 7 days of DigiQuilt use by these students, I used lessons learned in previous studies and decided that: the students would all use the software at one time, they would have challenge sheets nearly every day, they would be able to share their designs on printouts, magnets, and business cards for a larger percentage of their time using DigiQuilt, and their DigiQuilt time followed recess during which time three students helped me set up the computers. These students had the most opportunity to really dig in.

What the interviews told me

The student answers to the interview questions varied more between schools than between students at each school – students at CH participated in fewer structured activities outside of school than their counterparts at GW. The reason I think this difference might matter is that many factors could impact the amount of time a student would want to spend using DigiQuilt outside of the classroom, including access to computers that can run the software, time available to use the software, hobbies or experiences that relate to quilting or might make quilting meaningful, and more.

Besides the hobby and free time link, I was able to find out from students something about their confidence levels in math and art, as well as their opinions about who in their class was exceptionally good at math and art. These perceptions would likely play a role in helping a student decide whom to ask if they were having trouble in those subjects. In addition, it seemed likely that the students would be somewhat accurate in choosing students who seemed to almost give off an aura of being good at math or art.

How I used the Pre-test results

The students had a huge range of levels of success on the pre-tests. This data proved useful for choosing focus students and for situating the analysis of some episodes to a greater extent than would have been possible without the pre-tests. Since I had some information about things different children seemed to have a hard time with on the pre-tests, I was able to emphasize the role parts of the socio-technical system seemed to play within an episode. For example, if the pre-test showed that a student seemed to have a tough time comparing fractions when the denominators differed, then when a student attempted a challenge that was similar, I could point out not only that the student figured out how to solve it, but that it was something that might have been tricky for the student.

Choosing focus students

For each class, I chose several focus students. When choices needed to be made about whom to videotape or observe more closely, I leaned toward my focus students. The students did not necessarily know they were focus students. I still wanted to record a variety of students besides the focus students to make sure that I had captured the experiences of a wide variety of students, but I also wanted to balance that desire with having a more complete look at some students who I could predict would give me a nice range of data.

I chose my focus students to include boys and girls, students known for talents in math and art class, students of different races and ethnicities, and students with a range of performance levels in math. Gender, race, and ethnicity I considered obvious enough to forego asking specific questions, but for the other criteria I utilized information from the interviews and the fractions pretest. I also used teacher input informally for determining who was a strong math student or an exceptional artist. The last criterion was that the student needed to be somewhat willing to talk about their experiences. The interviews were a good opportunity for finding out which students were particularly camera-shy, less talkative, or less interested in discussing their problem solving.

Looking at GW4 data

Throughout the study, I was being particularly watchful for times when:

1. Learners were making connections between the abstract or symbolic concepts and the concrete examples they are creating
2. Children were showing that they cared about what they are doing – they were engaged, they were connecting their lives to their designs and their designs to their lives, they cared enough to persist in spite of difficulty, they added to the challenges and adopted new challenges on top of what I gave them

Following the chapter filled with day-by-day accounts of the GW4 DigiQuilt experience and a summary of some statistical data regarding students' use of DigiQuilt, I will present a chapter dedicated to describing my data analysis procedures in detail.

CHAPTER 5

SEMI-STRUCTURED CREATIVE PROBLEM SOLVING BY 4TH GRADERS USING A COMPUTATIONAL MANIPULATIVE FOR DESIGN AND MATH LEARNING

This chapter describes the course of events over seven days of DigiQuilt use, highlighting the experiences of several students in the GW4 classroom. The purpose of this chapter is to give an in-depth look at what happened in the classroom and what it looked like to use DigiQuilt. In the chapters that follow this one, I will present a more in-depth look at the data. Please note that throughout this dissertation, all names have been changed to protect the identities of the student participants. I will focus on the experiences of the focus students: Peter, Lisa, Emma, Imani, Edzier, and Joanna.

I chose these students because their experiences show a wide range of possibilities that can unfold during the use of this kind of computational manipulative – one that combines possibilities for designing artistic creations and exploring mathematics. Peter was chosen because many students named him as the best math student in their class. Lisa was chosen because her peers thought she was exceptionally artistic. Emma was chosen for her math skills and her willingness to discuss her problem solving strategies. Joanna was chosen partly for convenience – she was a helpful, cooperative, thoughtful student whose interaction style seemed to lend itself well to discussing strategies with other students. Imani was chosen because his math performance was generally low, but he seemed like he would take direction well and be willing to talk about and try new strategies. Edzier was chosen because of all the students, he had the most room for improvement on the pretest.

Day 1 – April fool's day – Getting started

The first day that the students in GW4 used DigiQuilt, their desks were arranged in groups of 4 so that each student was next to one student and across from the other 2 students (see Figure 19). This setup allowed the students to talk to each other in small groups, see the screen of one other child, and help each other when needed. Most of the students sat in their usual desks, but I moved one student so that two of the students I planned to focus on could both be recorded at the same time (Lisa was mentioned most often by her peers for her talent in art, and Peter was selected most often for being good at math). I was curious to see how these two students interacted with each other and what they would notice about each other's designs.

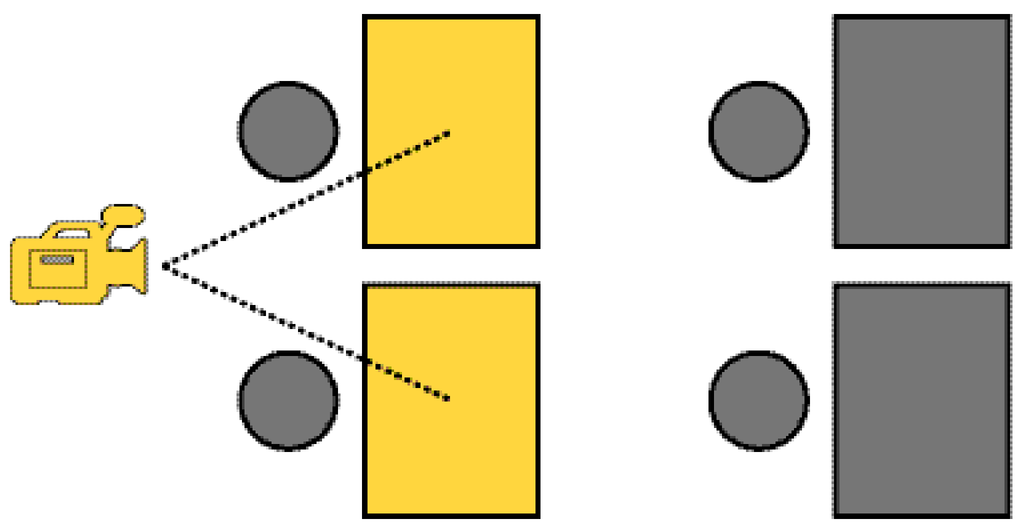


Figure 19. Diagram of the arrangement of cameras and student desks.

There were 5 challenges or tasks on the first day:

1. Make a quilt block that shows ½ and ½
2. Make a quilt block that shows ½, ¼, and ¼
3. Make a quilt block that shows 1/3, 1/3, 2/9, and 1/9

4. Make a quilt block that has at least one line of symmetry and shows the fractions from one of the problems you've already solved
5. Write the name of the design you made today that is your favorite.

Though there was no sound on my video recordings that day due to a technical error, video data suggests that the students were able to use the tools in the software with little instruction. I gave the students handouts with the challenges for the day. Most of the students worked through the challenges in numeric order, often showing their designs to neighboring students. Peter made 3 designs on the first day. The first design was half red and half blue in a diagonal striped pattern. For his second design, Peter figured out that he could overlap the pieces to make smaller shapes. He made a design that he named "5 squares" because it looked like 5 squares "on point" like diamonds (see Figure 20). The design is quite sophisticated – it has a sort of structural bilateral symmetry (though the colors would not match up if you folded it that way, the yellow parts land on pink and vice versa). His was the only design that used triangles and matched up in this way.

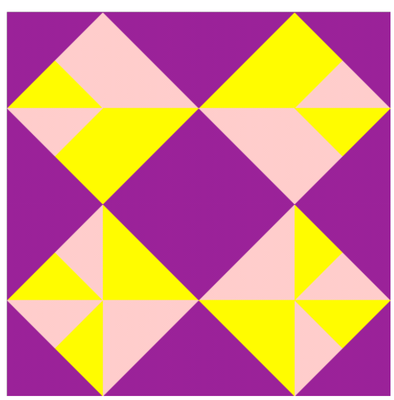


Figure 20. Peter's Design named "5 squares."

On the first day, the challenge that seemed the most difficult was the one that required switching to the 9-patch Block Work-Area. Some students tried for quite a long time to make 1/3 in the 16-patch work area, but that is impossible with the given pieces. About 38 minutes into their DigiQuilt time, Lisa, Peter, Emma, and Austin discussed something and look at Lisa's screen. It seems that they discovered that changing to the 9-patch quilt block helped them to make designs that showed the fractions in the challenge. A few minutes later, I showed the whole class how to switch to the 9-patch Block Work-Area. Within minutes, Peter constructed his design (see Figure 21) that he named "steelfire." This design was the first one that he was really excited about.

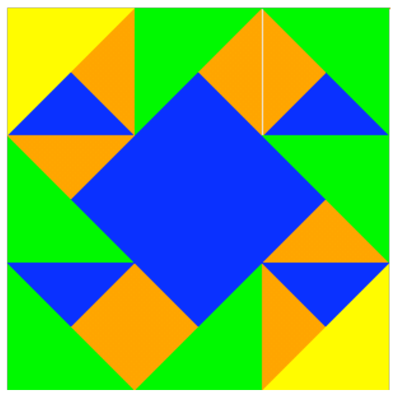


Figure 21. Peter's design named "Steelfire."

The students made 54 designs during their time using DigiQuilt. They worked for approximately 45 minutes. Table 3 shows all of the designs that each child created and saved. The students made between 1 and 9 designs for an average of about 3.2 designs each. That means it took the students approximately 15 minutes on average to make one design.

Table 3. Quilt block designs from day 1.

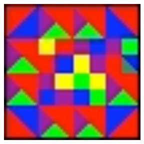
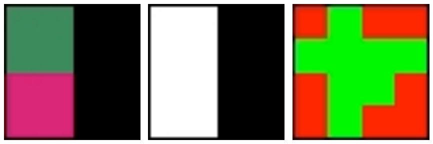
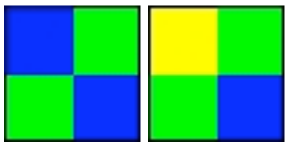
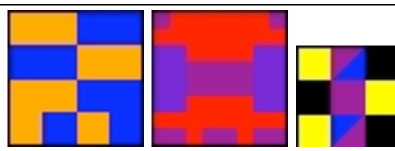
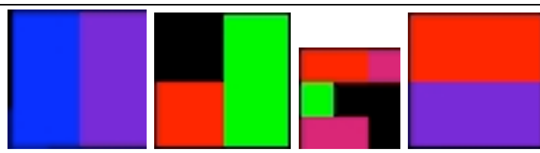
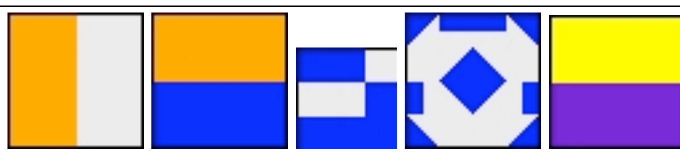
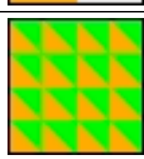
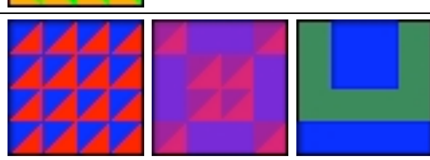
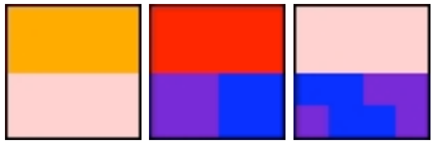
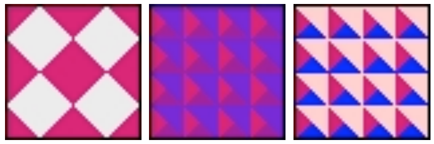
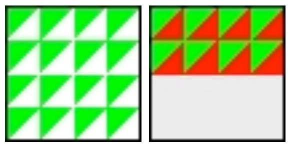
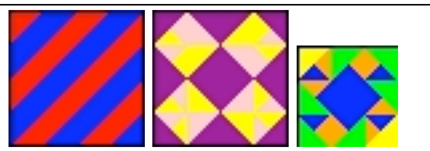
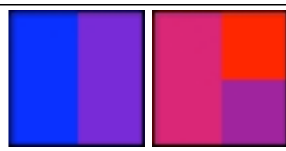
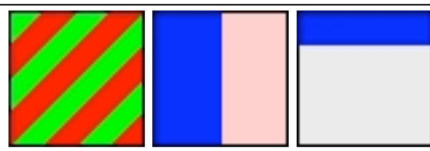
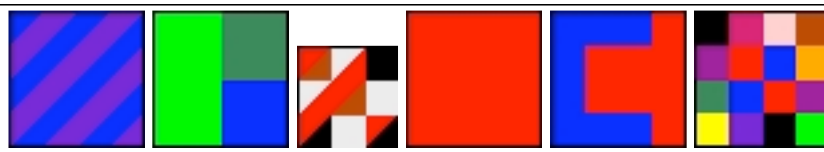
Asha	
Austin	
Beth	
Carter	
Douglas	
Edzier	
Emma	
Imani	
Jake	
Joanna	

Table 3 (continued)

Keyla	
Lisa	
Omar	
Peter	
Ramona	
Tyrel	
Wendy	

Day 2 - April 15 – Encountering symmetry

On this day, the students were able to get started quickly since they were already familiar with DigiQuilt. They started out by looking through printouts of their previous designs and showing their printed designs to their neighbors, discussing ones they liked and ones they didn't like as much. The challenges for the 2nd day were:

1. Make a quilt block that has exactly one line of symmetry.
2. Make a quilt block that has three colors and at least one line of symmetry. What fraction of the quilt block is covered by each color?

3. Make a quilt block that shows ½ and ½ AND has a line of symmetry.
4. Make a quilt block that shows ½, ¼, and 2/8.
5. Make the most interesting quilt you can think of.
6. Write the name of the design you made today that is your favorite.

Peter struggled almost from the start of the day to understand a challenge that was asking him to create a design with exactly one line of symmetry. Lisa used the select-a-grid to show Peter what was meant by this challenge. This is quite an amazing example because Peter was very confused about how a square could *ever* only have one line of symmetry, which isn't unreasonable since squares *do* have more than one line of symmetry.

Lisa tried to show Peter how her design only had one line of symmetry. First, she gestured from one side of her design to the other and talked about folding one side to the other. When this did not work to explain the challenge to Peter, she used a grid from the select-a-grid tool to show the line on her quilt block that was the line of symmetry, and then selected another grid to point out how that line did *not* work as a line of symmetry for her design (see Figure 22). Peter understood at that point. This discussion really seemed to help him understand how a design that is square could have only one line of symmetry, and the select-a-grid tool helped Lisa explain what she meant.

Peter's symmetry confusion

> Peter (to Lisa): "So wait, I don't get it. How can there be only one line of symmetry?"
> Lisa: "Hold on." She finishes working on her design.
> Peter: Continues to watch Lisa and says, "Oh, it's only half?"
> Lisa: "You can only fold it one way. All the other ways don't match up."
> Lisa completes her design as Peter watches. Her design has exactly one line of symmetry.
> Lisa displays the horizontal grid line and then the vertical grid line and tells Peter that the vertical would not work because the two halves wouldn't be equal.
> Peter: "Oh, now I get it!"

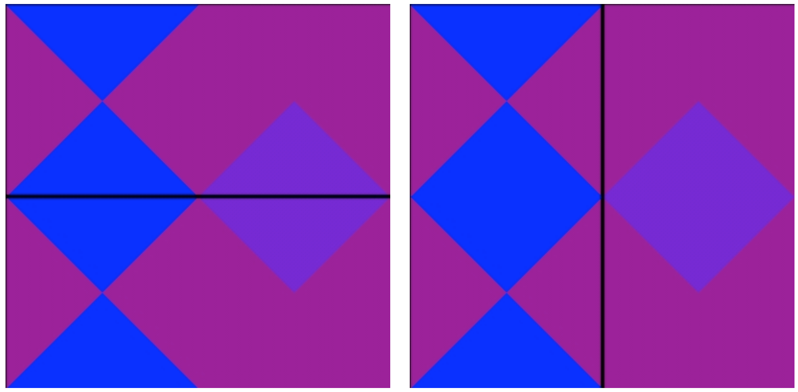


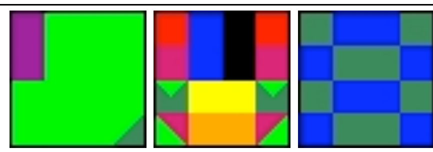
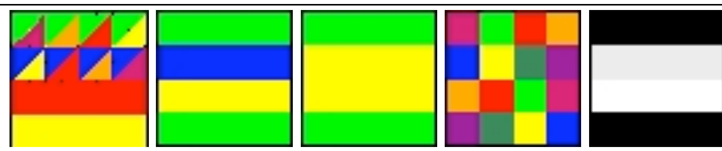
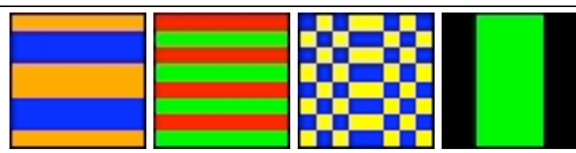
Figure 22. Lisa used two different grids from the select-a-grid tool to show Peter how her design had a horizontal line of symmetry and how the vertical line did not work as a line of symmetry.

In this example, Lisa had connected her understanding about the idea of symmetry to her design. In addition, she used a tool to illustrate the connection for another student's benefit. So, the select-a-grid tool not only helped Peter come to see the connection, it helped Lisa use her quilt block as a concrete example of "exactly one line of symmetry." In this case, the software seemed to support Lisa and Peter as they built a connection between the abstract idea of symmetry and the concrete examples they constructed. Within 10 minutes of completing the "one line of symmetry challenge," Peter is overheard reading a challenge aloud, "three colors and at least one line of symmetry. It's *so easy* to make a line of symmetry."

At one point, the children were comparing their progress and discussing different strategies or approaches to using DigiQuilt. Peter shared, "I try to make mine too complicated. That's what my problem is." Emma replied, "Well you see on DigiQuilt, once you're done, you can make them complicated." The students developed somewhat of a DigiQuilt identity – comparing and contrasting their design approaches with other students' approaches. They seemed to take pride not only in the product of their efforts (as evidenced by their sharing and discussing of designs at the start of the day), but in the process. Emma said, "How many did you do last week? I did like forty." Periodically, the students would check on each other's progress and tell how many designs they had made. The students shared their designs, their progress, and their plans. It was more common for them to share their designs than to work in isolation. They bragged about their accomplishments and praised the accomplishments of others. They compared and discussed each other's creations, noting similarities and differences.

On the second day using DigiQuilt, the students created 69 designs in about 40 minutes (an average of 4.1 designs each) (see Table 4). The students needed to be told several times when the time using DigiQuilt was over – they were reluctant to stop.

Table 4. Quilt block designs from day 2.

Austin	
Beth	
Carter	
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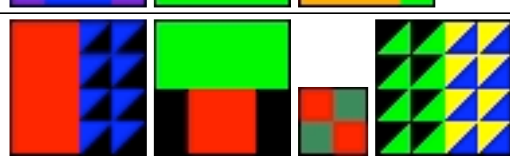
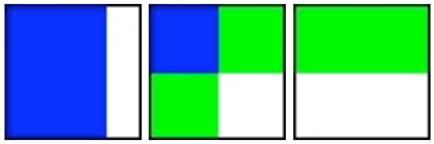
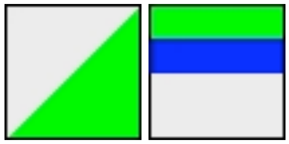
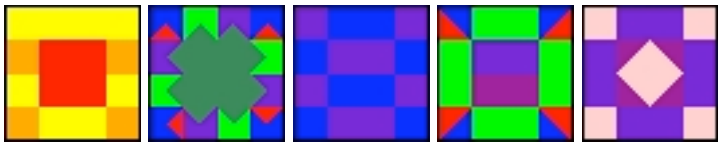
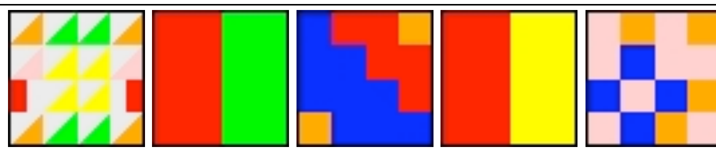
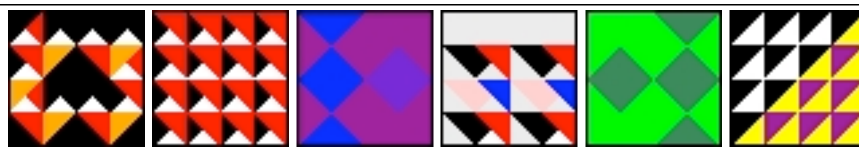
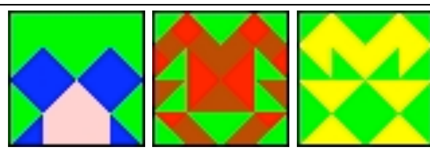
Edzier	
Emma	

Table 4 (continued)

Imani	
Jake	
Joanna	
Keyla	
Lisa	
Peter	
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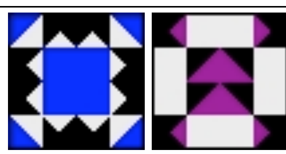
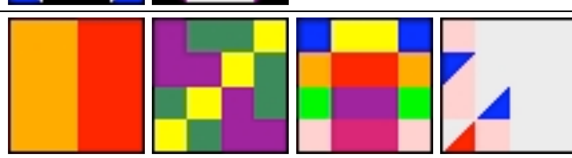
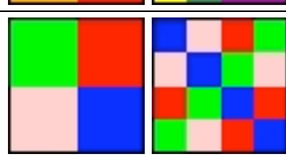
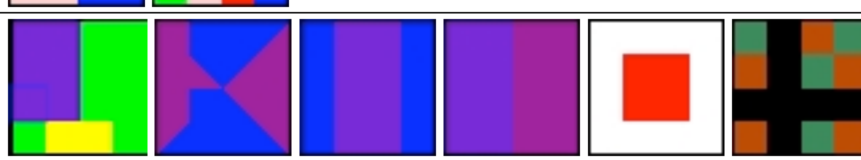
Serena	
Talisha	
Tyrel	
Wendy	

Day 3 – April 22 – Designs from the real world

On the third day, the students worked for about 35 minutes. The challenges from the third day were:

1. Make a quilt block that has exactly one line of symmetry and uses at least 3 colors.
2. Make the most interesting quilt block you can think of.
3. Make a quilt block that has 4 colors and at least one line of symmetry. What fraction of the quilt block is covered in each color?
4. Make a quilt block that shows 8/16 and 8/16 AND has a line of symmetry.
5. Make a quilt block that shows 8/16, 4/16, and 4/16.
6. Write the name of the design you made today that is your favorite.

Discussions about symmetry continued. I specifically requested that the students share their designs on the third day – something I had merely suggested previously. The students also continued to discover new things that they could do with DigiQuilt. Emma discovered the patch-swapping capabilities and showed Lisa. There seemed to be a raised awareness of the smallest shapes available for creating designs (artifact analysis suggests that only 2 students used small pieces on day 2, but 14 students used them on day 3). This allowed the students to create more detailed designs than before. The children carefully planned and considered how to go about creating quilt-block versions of things from real life. This type of careful evaluation and mathematization of the world took some real effort. It was not easy, but achieving designs that resembled something from the world resulted in admiration by peers and teachers alike. The children seemed to thrive on this sort of "hard fun." In this example, Emma, Lisa, and Beth discussed their designs and how to represent people.

Making people out of shapes

(16:50-18:10) Emma: “Yeah, I have the yellow face too.” “Who is that?”
Beth: “It’s you!”
Emma: “Oh, gosh.”
Beth: “It’s like you’re looking through binoculars.”
Emma: “How’d you get your hair like that?”
Beth: “You just do two rectangles and then you put a square.”
Beth shows Emma (on Emma’s computer) how to make the hair (see Figure 23). In the process, she also shows Emma where the rectangles are. Emma gets Lisa’s attention to show her how to find the rectangles too.

(18:40) “You guys have pathetic things. You guys don’t have the real shapes.” Emma tells Peter and Austin. She and Lisa tell the boys in front of them how to get all of the shapes (tell them how to switch from “basic” to “all” mode). They talk for a moment about how neat it is.

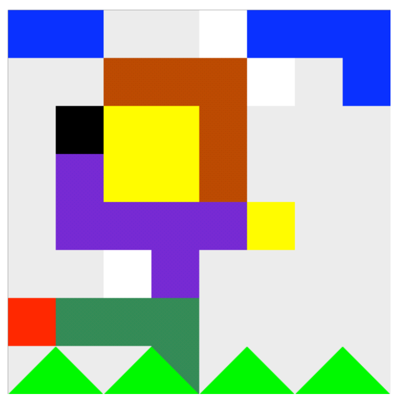


Figure 23. Beth’s design depicting Emma – entitled “Googie.”

Emma seemed particularly excited about creating designs that were representative of objects in the real world. She created a design that she said looked like a hand holding a can of Dr. Pepper and a smiling face with big eyes. Emma was not alone in her goal to create things from the world. Peter named one of his designs “Ballerina,” Beth made a portrait of Emma (discussed in the previous example), and Douglas created a skydiver design.

One particular example of constructing a design based on something from the real world was when Joanna designed her first "house" quilt block (see Figure 24). Joanna and Keyla spent a fair amount of time discussing their designs, looking for advice from each other and thinking aloud about things that are needed. On this day, Joanna spent a long time trying to get the details right and requesting input from Keyla.

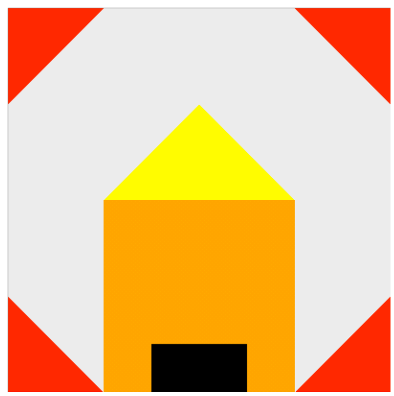


Figure 24. Joanna's design named "House."

Joanna's "house"

(5:00 – 16:27) Joanna is working on a quilt block that ends up looking like a house.

(7:40) Joanna gets Keyla's attention and shows her the 'house' design. She explains what it is and points out the roof and door. Keyla asks her how she accesses the mode with all of the shapes and she shows her.

(8:00) Joanna explains to Keyla that the red triangles in the corners are supposed to make it look like a picture in a scrap book.

(8:25) Joanna experiments with putting clouds in her picture. They are rectangular.

(10:01) Joanna tries to find a way to put a sun into her picture.

(10:27) Joanna asks Keyla what else she should put in her picture, "Keyla, can you think of anything else I need?" Keyla replies something about not knowing what the blue rectangles are. Joanna says that she knows it doesn't really look like it, but that those are clouds. Keyla says a sun, and Joanna asks for suggestions as to how to make a sun. They collaborate for a while to come up with an idea for a sun. "Maybe take one of those little squares and put it" "put it right here" "yeah, so it looks like it's behind the clouds." "Ok, yeah, good idea. Thanks"

(11:04) Joanna says something about adding a tree, which she then tries.

(11:42) Joanna says, “Keyla, look, it’s a tree. It doesn’t really look like it, but oh, well. I’m going to do the same tree on the other side.” Once she adds the tree to the other side, Keyla suggests that it looks like a forest. Joanna decides to take the trees out because, “I really wanted them to look like trees.”
(12:15) Joanna tells Keyla that she decided to name her design 'house'.
(13:09) Joanna says she wants to add flowers to her picture, but can't figure out how to draw them with the given shapes.
(13:42) Joanna decides to take out the trees and the sun/clouds because they don't look realistic.
(15:10) Joanna tries to put windows onto her house, but struggles. She sighs with frustration.
(15:42) Joanna decides to keep her design the way it was originally. She tells someone that she calls it house because it looks like a house, but that it doesn’t *really* look like a house.
(16:14) Joanna saves her first design as 'house'.
(16:27) Joanna says: "Miss S, come look at mine!" referring to her house design.

Joanna put a lot of time and effort into her “house” design. She did not give up even when she seemed frustrated. In the end, she was eager to share the result with her teacher and seemed quite proud.

On the third day, the students created 89 designs in about 40 minutes (see Table 5). That means the average number of designs per student was 4.9 and the average amount of time to make a design would be around 8 minutes.

Table 5. Quilt block designs from day 3.

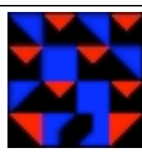
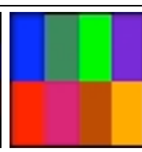
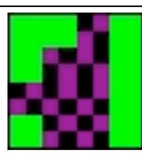
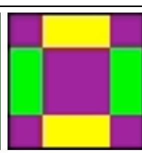
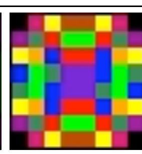
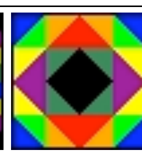
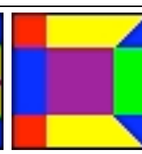
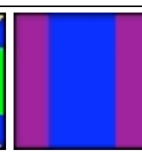
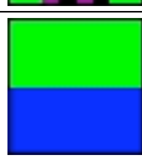
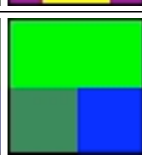
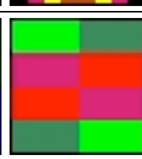
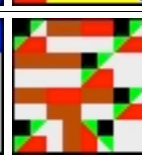
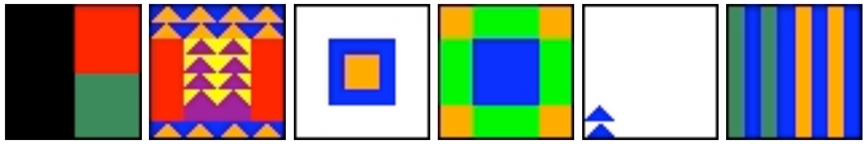
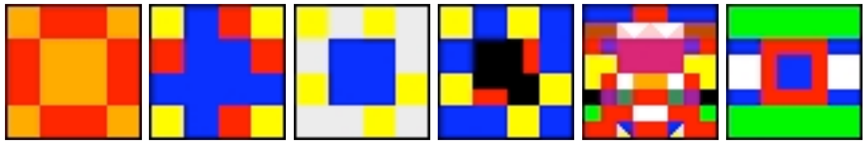
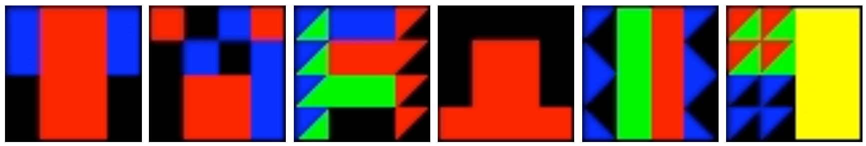
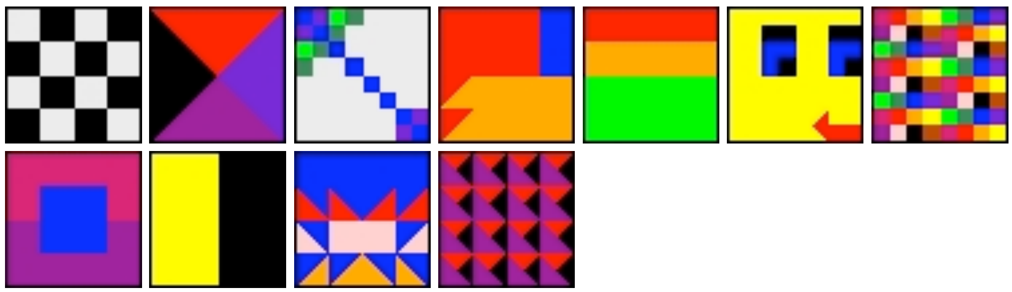
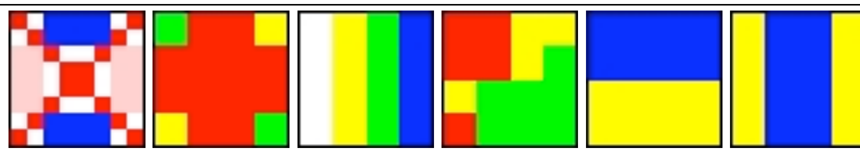
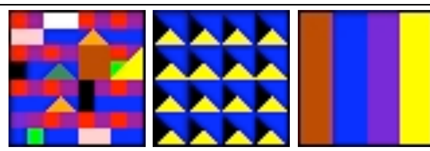
Asha	  
Austin	     
Beth	    

Table 5 (continued)

Carter	
Douglas	
Edzier	
Emma	
Imani	
Jake	
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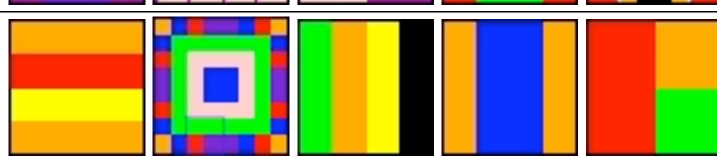
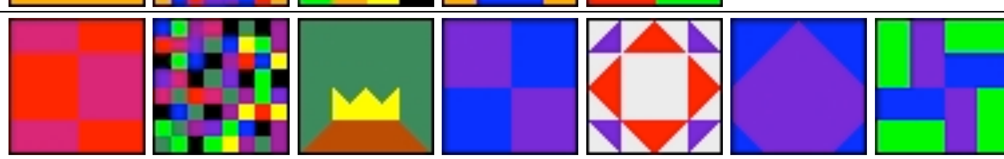
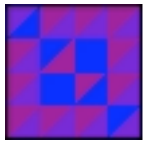
Keyla	
Lisa	
Peter	

Table 5 (continued)

Ramona	 
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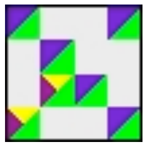
Talisha	  
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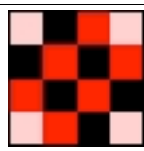
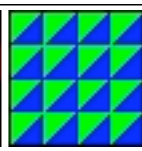
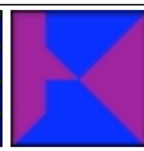
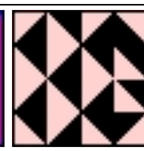
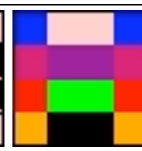
Wendy	    

Day 4 – April 29 – Tricky fractions

By day four, the students are able to get started right away making designs that solve the challenges on the sheet:

1. Make a quilt block that has at least one line of symmetry and uses at least 3 colors.
2. Make the most interesting quilt block that you can think of.
3. Make a quilt block that has 4 colors and exactly one line of symmetry. What fraction of the quilt block is covered by each color?
4. Make a quilt block that shows 15/32, 1/16, and 15/32 AND has a line of symmetry.
5. Make a quilt block that shows ½, ¼, and 2/8.
6. Write the name of the design you made today that is your favorite.

On the 4th day, the theme that stands out the most is the struggle to figure out the fraction 15/32. In particular, Emma and Austin struggled in parallel with this fraction. The desks were arranged differently for standardized testing (see Figure 25). It is not too surprising, then, with standardized testing on their minds that there seemed to be more of an awareness of "copying" than usual. Several students brought up copying in reference to DigiQuilt use that day,

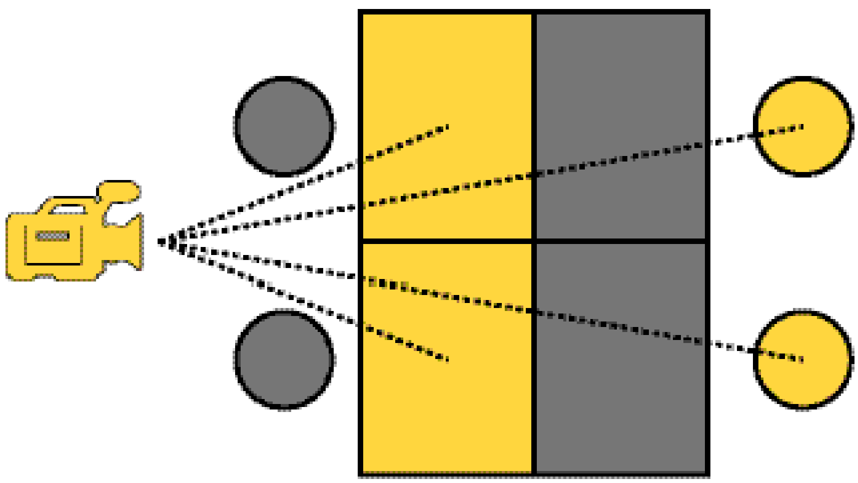


Figure 25. The desk arrangement for standardized testing meant that all the students' desks were facing the front of the room, but for DigiQuilt use every two rows were pushed closer together.

The students' familiarity with some algorithms for finding equivalent fractions was pushed to the limit by 15/32. Emma worked over the course of about 7 minutes to solve the challenge: "Make a quilt block that is 15/32 some color." She struggled to, first, figure out if 15/32 was a reduced fraction or not. She worked initially by asking her friend Lisa, but after trying a few things unsuccessfully, she got the teacher involved too. The teacher guided her thinking and encouraged her to explore her ideas. Eventually, she used the fractions-feedback to figure out how to cover 1/32 of the quilt block with a particular color and successfully solved the challenge. This exploration took place over a long period of time (7 minutes) relative to the amount of time it takes to actually create a design once the fractions are understood and the total amount of time available to use DigiQuilt in one day.

Emma's fraction exploration

(20:51) Emma (to Lisa): "What's another name for 15/32s?"
Lisa and Emma have a conversation about this. They write some guesses and calculations on paper.
Emma: "How many [DigiQuilt] squares is that?" (Emma counts and mutters.) "Because that's the question that I'm on."

(21:49-23:13) Emma is placing big squares in a diagonal pattern in the block work area. After she places the 4th square, she says, "1/4," which she seems to be reading from the color button on her screen. She continues trying to get 15/32 by adding shapes to her screen, but she is not successful.

(23:13) Emma: "Lisa, I'm trying to understand this thing, but I don't get it." Emma asks Lisa again about the fraction. Emma adds, "I got nothing," expressing her lack of understanding. Lisa and Emma try to figure out another "name" for 15/32.
Lisa: "Maybe it's 1/16." Emma adds some squares to her block work-area and says she doesn't think so because then that would be "it." Emma then suggests maybe it *is* 1/16 and asks, "What's another name for 1/16?" Lisa writes on paper and tries to figure it out and comes up with 2/32 and 4/64. Emma continues to struggle with the fraction. Lisa goes back to work on her task (she is not working on the same challenge as Emma).

(25:24) Emma says, "Lisa, it's not on here." She keeps adding pieces to her block work area and then reading the fractions from the fractions-feedback aloud.

(25:52) Emma: "It can't be 2/32"
Emma asks the teacher: "What's another name for 15/32"?
Ms. S asks the girls if they know a way to find an equivalent fraction.
They both answer, "divide it". This conversation continues. Eventually, I clarify that you can't *reduce* it anymore (the conversation includes some mention of the idea that you can still come up with another name for it by multiplying).

(26:48) Emma and I discuss how 15/32 is close to 1/2, but not quite the same.

(27:36) Emma has definitely figured out a strategy for finding 15/32. She is adding rectangles to her screen one at a time. Since each one is 1/32, once she has 15 of them, she finds the answer (see Figure 26).

(27:56) Emma: “I found it!” (She hides her computer screen.) “I'll tell you what it is.” Emma covers her screen and tells Lisa she can't see it because Emma doesn’t want her to. Emma added that she, “[doesn't] want Lisa to know.” (Emma doesn’t want Lisa to see the design because she wants to explain to her how to solve it rather than just letting her see.)

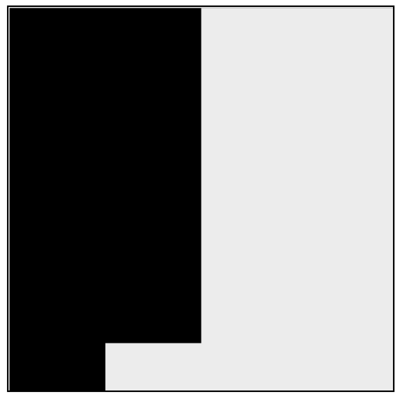
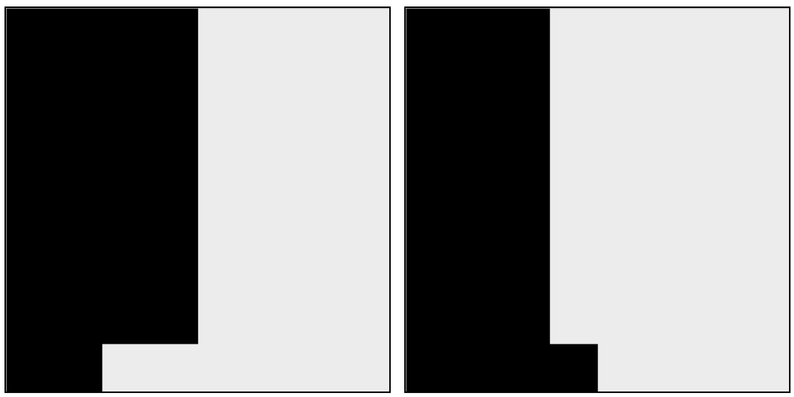


Figure 26. Emma’s 15/32 design that she created but did not save.

Emma was able to solve the challenge with the support of Lisa, her teacher, and myself. She did not give up even though she had a tough time. The feedback from the software helped her find a way to solve the problem eventually, but it was not the only source of support as Emma struggled to make this connection between a concrete example of 15/32 and the symbolic fraction in the challenge. Emma used a variety of strategies to find a solution for the problem – some methods she recalled from her previous experiences with fractions, some support from friends and teacher figures, and some support from the software.

It was surprising how long Emma persisted in attempting to solve this challenge in spite of her difficulties since she normally took such pride in solving all the challenges quickly. This effort altered her normal productivity level a lot – she only saved 3 designs that day, while her average over the course of all 7 days using DigiQuilt was 8 designs per day. In the end, she did not even save the design that solved the “15/32” challenge.

Later that same day, Lisa attempted to solve the 15/32 challenge and was not successful. However, she seemed to be able to convince herself that she had correctly solved the challenge based on her experiences with Emma. Lisa used a pencil and paper to do a quick calculation and seemed satisfied that her design showed 15/32. To view the designs side by side, see Figure 27.

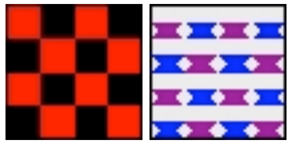
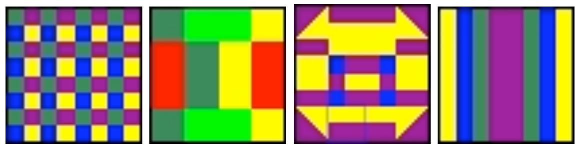


(a) **(b)**

Figure 27. Emma's design (a) and Lisa's design (b) for 15/32.

On the 4th day, the students created 54 designs in about 40 minutes (see Table 6). So, on the 4th day, the students made an average of 3.2 designs each. This change of pace might be attributed to the difficult fractions, or perhaps the introduction of a new activity. That day was the first day that the students were told about the opportunity to request business cards and magnets with their designs on them, so the students spent some time choosing designs to have printed specially. Some students continued working for up to 10 minutes after they were told to shut down their computers for the day, so they had a little more time than others. They wrote down designs that they wanted to have on business cards or magnets.

Table 6. Quilt block designs from day 4.

Asha	
Austin	
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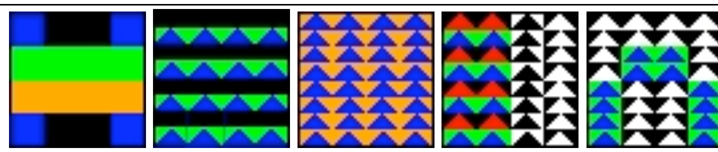
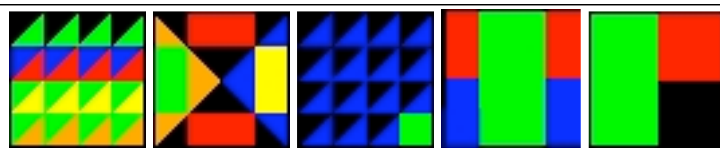
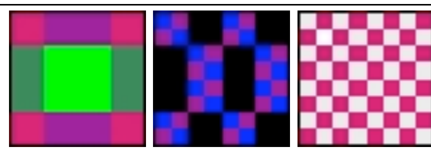
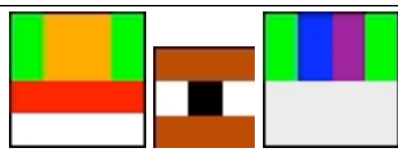
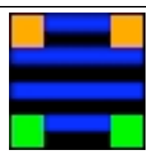
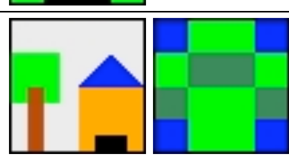
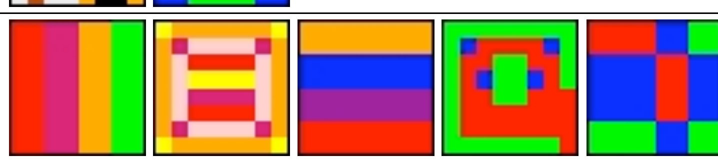
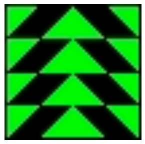
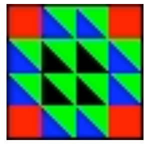
Carter	
Edzier	
Emma	
Imani	
Jake	
Joanna	
Keyla	
Lisa	

Table 6 (continued)

Omar	  
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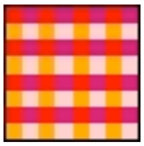
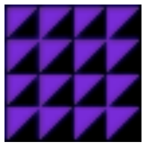
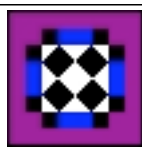
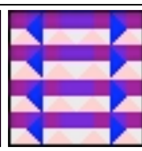
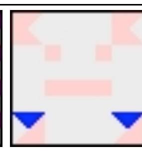
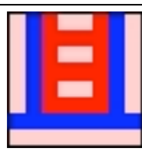
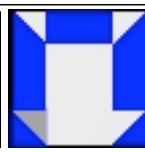
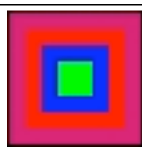
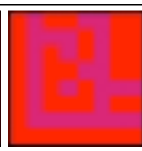
Ramona	 
Talisha	   
Tyrel	 
Wendy	 

Day 5 – May 6 – The substitute teacher

Day 5 began on a sour note that I think may have impacted the students' ability to concentrate and talk to each other. This is the day that this class had a substitute teacher who began the time we were supposed to be using DigiQuilt by lecturing the class about how she was tired of some of the "attitudes" in this class. I was surprised because, in all of my experiences when their regular teacher was present, these students behaved very well. Of the 4 classrooms of children I had the pleasure to work with, this group was the most orderly and task-oriented. It seemed as though because of the way their day was going, the kids took a long time to get comfortable talking to each other that day in class, and they did not interact the same way as usual.

The challenges for the day were:

1. Make a quilt block that has one line of symmetry and is 5/16 some color (fill in the rest of the design with other colors).
2. Make a quilt block that has 3 colors arranged however you'd like. What fraction of the quilt block is covered in each color?
3. Open a quilt block made by somebody else and add or change something to make it look different. What fraction of the quilt block is covered in each color?
4. Circle the bigger fraction: 5/32 OR 5/16. Make quilt blocks to show someone how you know which one is bigger.
5. Write the name of the design you made today that is your favorite.

There was a lot of sharing of "neat" designs on this day that seemed to be prompted by the challenge to alter a quilt block someone else made.

We predicted that some children would need help understanding that a bigger denominator means that each piece is smaller, so we included a few challenges that were designed to highlight that relationship between fractions. This episode shows Imani struggling with the challenge that asks which is bigger: 5/32 or 5/16. Imani filled in 5/16 of his block work area with one color, but the rest was blank. He and I walked through a way to solve the problem, talking about fractions along the way.

Imani decides which is larger

> (10:49) Imani tells me he is working on a design that shows 5/16 and 5/32. Imani: "This says, 'show somebody how, which one, how 5/32 is bigger than 5/16.'" The challenge asks which one is bigger and to then make some quilt blocks that could be used to help someone else how you know which one is bigger. Imani has already decided which is bigger, so he is setting out to show that 5/32 is bigger. "This is 5/16." [points at the design on his screen]

Imani clears his block and starts over. He is building on the 16-patch block work area, and begins adding large squares, one at a time. I point out, "there's 4/16, 5/16, 3/8," as Imani adds more large squares.
KK: "Have you seen any thirty-seconds yet? How big do you think 1/32 is?"
Imani: "Umm…" [pauses].
KK: "What does 1/32nd mean?"

On his pre-test about fractions, Imani correctly answered the question that tested the understanding that when the numerators are the same, a bigger denominator indicates the smaller fraction. The reason he wrote on the pretest was, "you don't have to break in small pieces." In spite of that understanding, Imani seems to struggle with this challenge.

Imani opens up the block browser and I suggest that he open up a certain design. Imani opens that design and I point out that, on this design, indigo takes up 1/32 of the total quilt block. I point out that the indigo is a triangle that is half the size of a patch, so a triangle of this size is 1/32 of a quilt block. The student states that he will need to add five triangles of this size to make up 5/32. This is incorrect, and I ask him to repeat what he just said. Imani realizes he is incorrect, and begins building on a blank block rather than adding to the existing design. He adds five triangles to get 5/32.

KK: "So, what can you tell me?"
Imani: "So, 1/16 is bigger than 5/32."
KK: "5/16 or 1/16? Why don't you tell me again what you just said."
Imani: "1/16 is greater than, wait, 'cause…" [He begins placing shapes into the block work area.]
KK: "There's 1/32, 2/32, 3/32, 4/32…"
Imani: "Huh. Um, hang on." [He moves his mouse to the 'clear' button and presses it.]
KK: "That's okay, don't clear. Keep going, click on 'no' for now. Add one more." [I cover up the feedback o the screen,] "Ok. How much do you think is covered in indigo?"
Imani: "5/32nds. [Pause.] So, 1/16 is bigger than 5/32nds."
KK: "I want you to concentrate again and say that one more time. Think to yourself what you're gonna say."
Imani: "Um, 1, not 1-si… 5/16ths is greater than 5/32nds."
KK: "Ok. So, maybe your instinct was a little bit different." I go on to note that sometimes intuition about fractions can be wrong, and that building a design can help you find the correct answer.

Since this has taken awhile, I ask Imani to explain how he figured out which is greater out of 5/16 and 5/32. He talks about how he opened up another design to figure out what piece takes up 1/32, and continues to explain how adding pieces to the blank block work area helped him figure it out.

Imani made some big strides in understanding how to use the manipulative as a way to think about the challenge and understand it. The fractions-feedback reflected the changes Imani made to his concrete design in a symbolic way. The software helped him in this regard, but the reflection on what he was doing also played an important role.

Not every student was able to succeed in every challenge they attempted. This next episode took place over about 15 and one half minutes. Edzier struggled with his design for the first challenge: "Make a quilt block that has one line of symmetry and is 5/16 some color (fill in the rest of the design with other colors)." He asks for help several times. He seems to understand the goal, and strive to achieve it for most of the time, but in the end his design does not successfully solve the challenge.

Edzier's double challenge

> [GW4-5-06-04-t1][EDIFF][Fractions]
> (0:00) Edzier works on the first challenge. His design looks like the letter "h". He raises his hand after he finishes. He shows me his design. He says he doesn't get number 1. I tell him that it looks like he has 8/16 red and 8/16 yellow, but he needs to have 5/16 of some color. Edzier changes his design so that it is 5/16 red, 5/16 yellow, and 3/8 green. His original design was symmetric, but now it is not. What began as a difficulty with fractions turns into a difficulty with symmetry. Edzier continues to try to solve the problem.
>
> (6:00) Edzier clears his design without saving. He puts in 4 green squares and raises his hand. He adds more shapes. He raises his hand again and then clears his work again without saving.
> (8:40) Edzier raises his hand again. He glances at the camera. He looks either very tired or very frustrated. He still doesn't get it. I ask him if he wants to skip it and come back to it, and he shakes his head. I ask, "How much is covered now, with the green color." Edzier replies that 1/16 is covered. "Do you think there is a way you could cover 5/16 with green?" Edzier puts in 4 big squares. I explain that he has to be careful when he puts in the last 16^{th} because he needs to keep a line of symmetry. He tries to set a patch-sized green square overlapping the edges of two snaps in the block work area, which doesn't work. I tell him he'll need to use two smaller pieces if he wants to put it there. At this point, I think Edzier will get it soon, so I leave him to help other students.

(10:47) Edzier has selected a grid that splits his block work area into 8 pieces and has a vertical line (seems to be the line of symmetry he is after). He repeatedly picks up the same green square and tries to set it on the line, but it always snaps one way or the other. Then, he uses smaller rectangles (but since he already has 5/16 of the quilt block covered in green, this does not help him solve the challenge). He raises his hand to ask for help again. He needs to remove the piece that won't land where he wants it before his new idea will work.

(10:52) I ask him, "How many 16ths are covered in green." He replies, "3/8." I ask, "How many 16ths it that?" He answers correctly. He removes both of the green squares from the bottom row, and as he does that, I point out that when there is just one of those squares left, it was 5/16 in the fractions-feedback part of the screen. I suggest that he use 4 small squares to fill in his design in a symmetric way. He adds the squares in a way that maintains the symmetry.

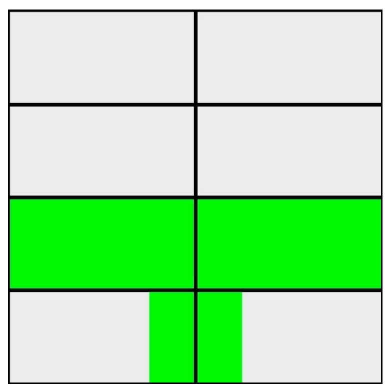


Figure 28. Edzier's unfinished design solves both parts of the challenge, but leaves blank spaces.

Unfortunately, his design still has blank spaces that contain no pieces. I suggest to him that he fill in the rest of his designs with other colors.

(11:40) He fills in the rest of his design and saves it as "design1", but the way he fills it in is not symmetric, and it includes green, which makes it so that the quilt block is not 5/16 green anymore. (See Figure 29 - the design only included the solid green squares before he filled in the remaining patches.)

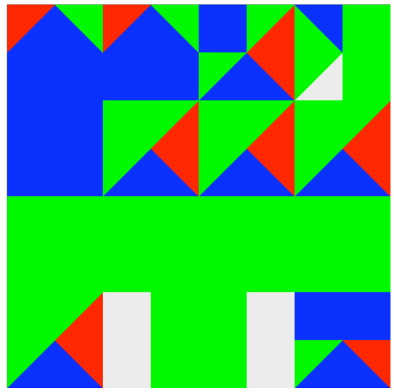


Figure 29. Edzier's design after he fills in the rest of the patches.

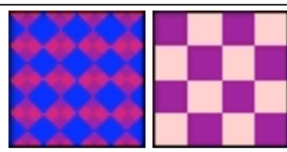
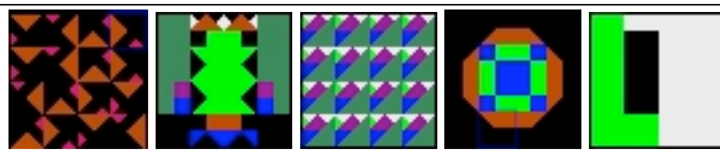
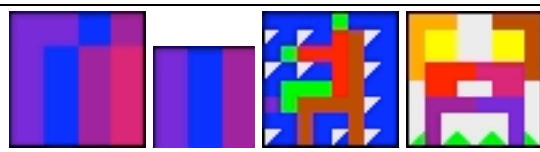
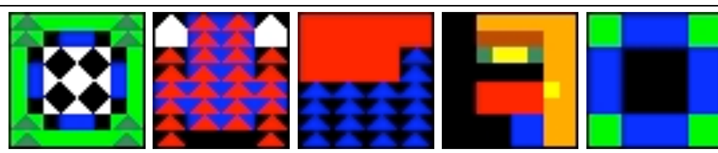
> (15:23) Edzier raises his hand for a moment. Then, he saves his design and clears it. It was not symmetrical, and it did not have the right fraction anymore. In my opinion, he looks very tired on the tape, and I think he honestly just wanted to move on to something else. The fractions-feedback would not have been telling him he had 5/16 green, so he probably made a conscious decision to move on in spite of the design not quite solving the challenge.

Getting children to save several versions of the quilts as they work through them might be one way to help them become less attached to any one solution. Perhaps if Edzier had several examples that contained 5/16 green and several that were symmetric, he would be able to figure out how to make one design that would have both attributes. In actuality, although his final quilt design did not solve the challenge, it is quite possible that he was able to connect his quilt block with each of these concepts in turn and just not able to make one quilt block that embodied both abstract ideas. The way he was attempting to make his design symmetric suggests that he realized how much green he needed to add, but that he was unable to understand how he could add that amount of green in that location (a location that would maintain symmetry).

At the end of the class period, several students gathered by one desk and showed each other their business cards. Wendy expressed her pride that someone asked for a business card of one of her designs (they requested it directly from me rather than taking their chances with possible trades at morning "social" time). Wendy told me about how the students had been trading their designs during their break time. She showed me some of her favorite designs she had collected.

The students' time using DigiQuilt was just a little bit shorter on the 5^{th} day since they started later than usual. In the 35 minutes of work time, they created 73 designs (Table 7). That means that students each made an average of 3.8 designs on the 5^{th} day (about 9 minutes per design).

Table 7. Quilt block designs from day 5.

Asha	
Austin	
Beth	
Carter	
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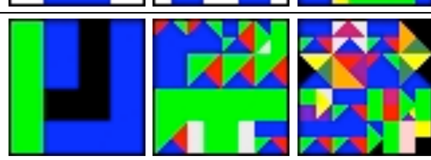
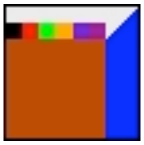
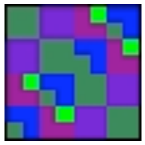
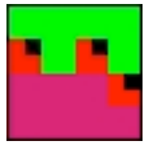
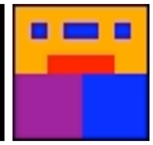
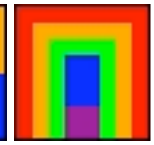
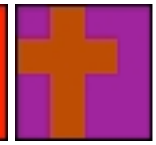
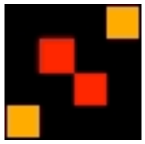
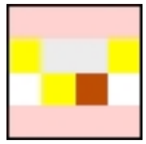
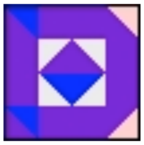
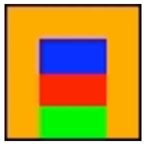
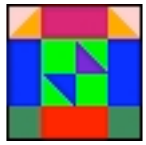
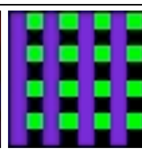
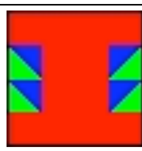
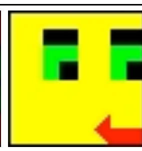
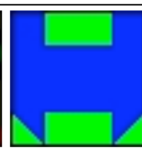
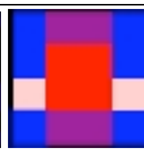
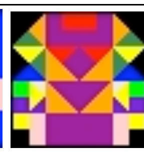
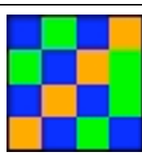
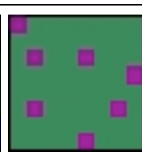
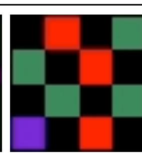
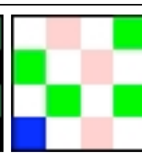
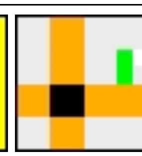
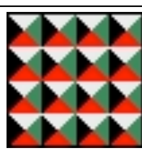
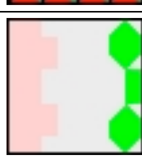
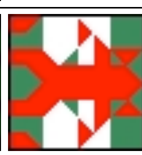
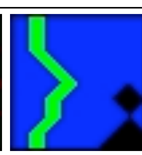
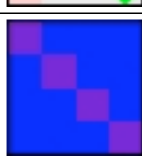
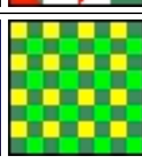
Edzier	

Table 7 (continued)

Emma	          
Imani	   
Jake	 
Joanna	  
Keyla	  
Lisa	       
Omar	
Peter	   
Ramona	 

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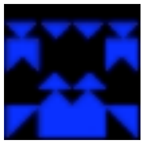
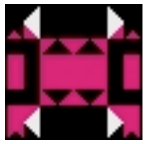
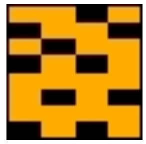
Serena	  
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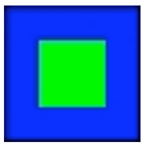
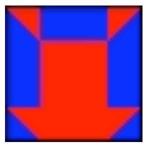
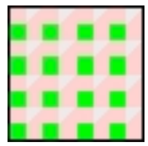
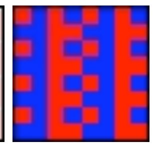
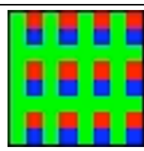
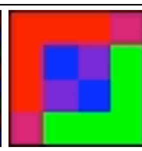
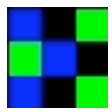
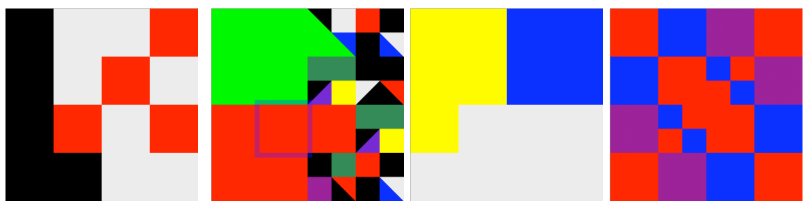
Tyrel	    
Wendy	  

Day 6 – May 13 – Gearing up, Winding down

On the 6th day, an undergraduate student came with me to the classroom to help out. She and I both were available to students to answer questions and assist them in their activities. On the 6th day, the challenges were:

1. Make a quilt block that has one line of symmetry and is 7/16 some color (fill in the rest of the design with other colors).
2. Make a quilt block that has 4 colors arranged however you'd like. What fraction of the quilt block is covered in each color?
3. Open a quilt block made by somebody else and add or change something to make it look different. What fraction of the quilt block is covered in each color?
4. Circle the bigger fraction ¼ or 5/16. Make quilt blocks that could show someone how you know which one is bigger.
5. Write the name of the design you made today that is your favorite.

The students completed their challenges without a lot of need for extra help from me. The children wrote more on their sheets than they often did in the past. This might have been a result of having an extra adult researcher present who could help encourage them to write down their fractions when the challenge sheet requested it. The students also seemed a bit more confident in their creation of designs that were meant to help someone else see which fraction is bigger. There was a greater variety of solutions for this type of problem (Figure 30). (Note: The design in Figure 30b does not solve the challenge exactly, but it is the result of a student's efforts with that challenge.)



(a) (b) (c) (d)

Figure 30. A small selection of quilt blocks that show ¼ and 5/16.

By day 6, the students seemed particularly interested in making and collecting designs that they wanted to have printed on business cards or magnets. After all, they were getting closer to the end of the school year and didn't have much DigiQuilt time in school left. They were still very excited about the prospect of having magnets and business cards with their quilt designs on them. The 5th graders at their school had sold some of their magnets at their school concession stand. This was cause for great excitement amongst the 4th graders – somebody might actually want to *buy* their designs.

By this time, some of the students had taken copies of the software home and were using it on computers outside of school. I offered the students who were using the software at home some challenges they could use if they wanted. All the students who were using the software at home raised their hands in order to get a challenge sheet. Austin's little brother was also interested in using DigiQuilt and printing business cards, so I told him how to go about doing that with help from his parents.

Day 6 was the second time I gave students a challenge that involved starting from a previous design and creating something new. Some students seemed to really enjoy that type of challenge (see Figure 31 for several before and after designs from day 5 and day 6). For example, Beth used several designs as a starting point that day, which was the day she saved the most designs. Several of these designs were largely unchanged – for example, changing the eye color on the face design (Figure 31f) or adding a star to the cross design (see Figure 31h).

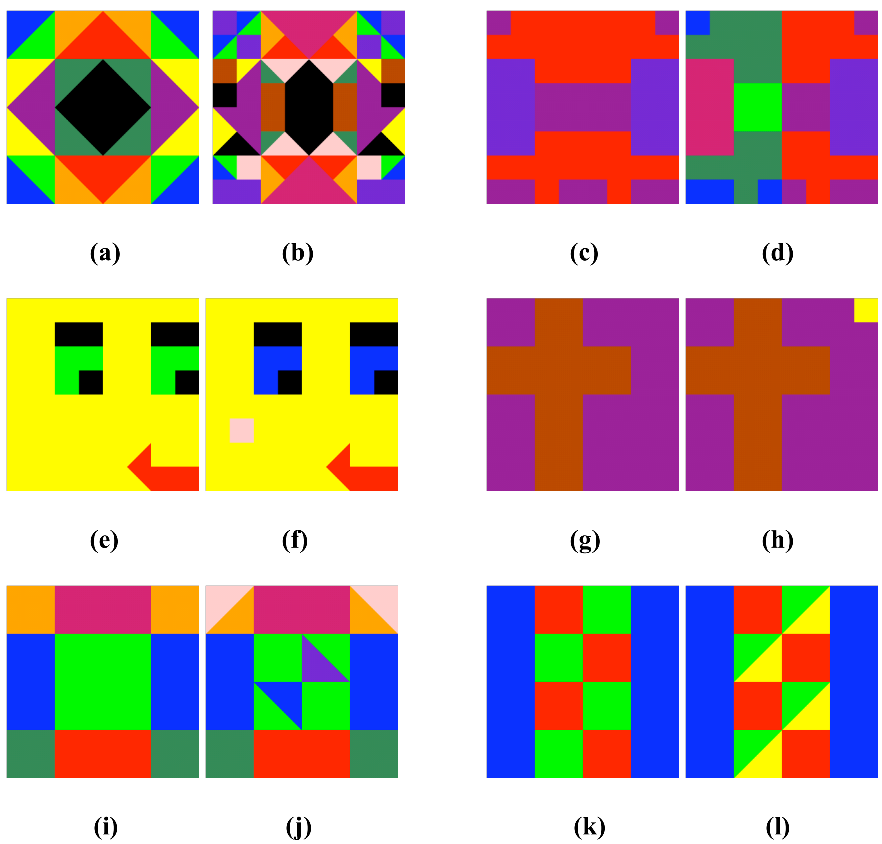
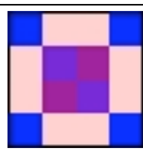
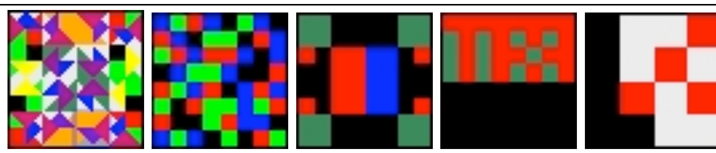
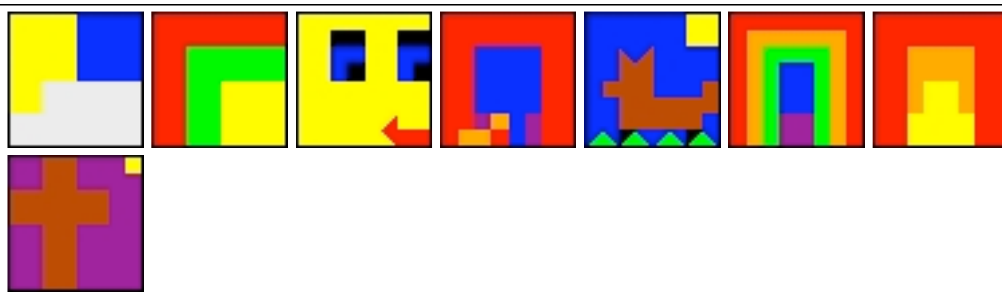
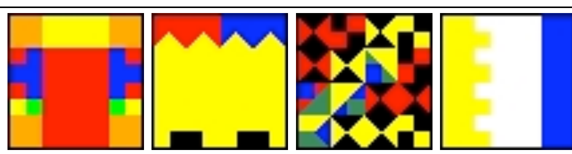
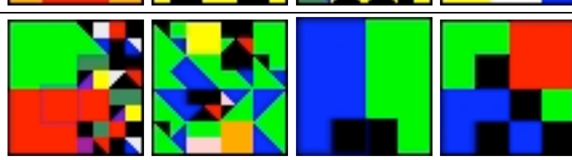
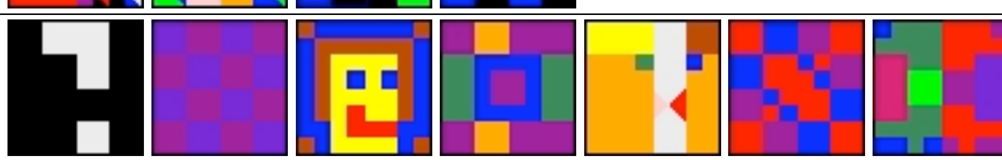


Figure 31. A series of images that students chose to alter (before and after).

On the 6th day of DigiQuilt use, the students created 75 designs in about 40 minutes (about 4.4 designs per student, so approximately 9 minutes per design) (see Table 8). I left early that day, so an undergraduate student assisted me by collecting the challenge sheets and business card and magnet requests. From the video data, it appeared that the students were a little bit better about ending on time that day than they had on some previous days. Still the process of putting away laptop computers in a cart after shutting everything down did take a few minutes.

Table 8. Quilt block designs from day 6

Asha	
Austin	
Beth	
Douglas	
Edzier	
Emma	

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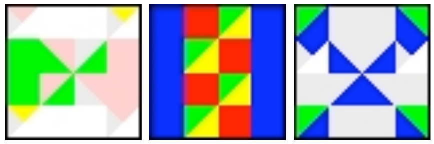
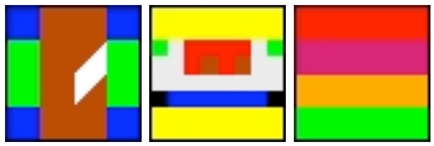
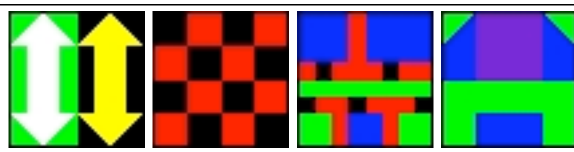
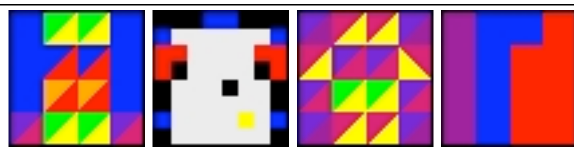
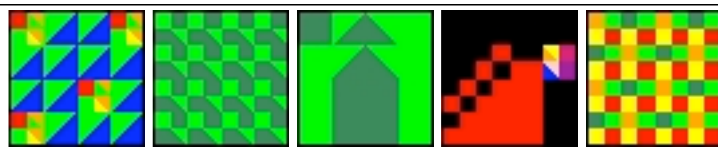
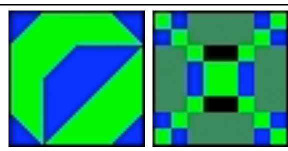
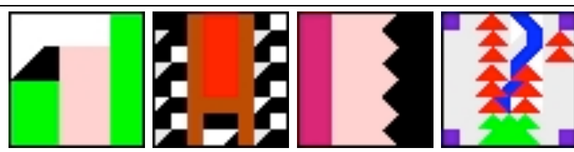
Imani	
Jake	
Joanna	
Keyla	
Lisa	
Omar	
Peter	
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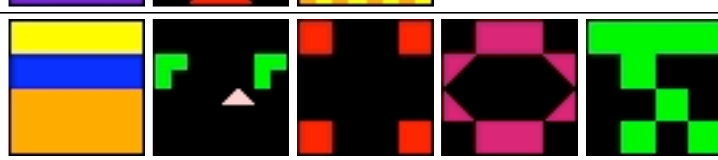
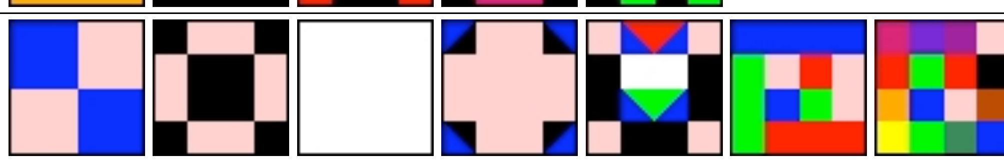
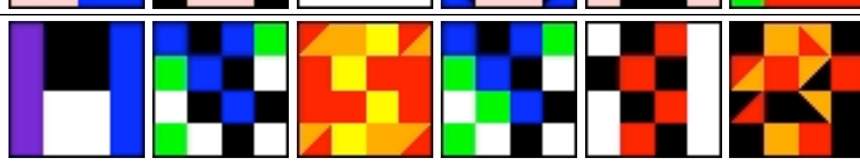
Serena	
Tyrel	
Wendy	

Day 7 – May 20 – Wrapping up the school year and DigiQuilt time

The last day that the students used DigiQuilt was very close to the last day of school. The classroom did not look at all like it did in previous weeks – many decorations had been removed, student work had been taken home, and desks were empty or nearly so. In addition to the regular summer anticipation that comes with the end of an academic year, the school was in a state of transition since all of the teachers would be changing classrooms or possibly schools in the coming fall. For some of these 4th graders, next year would be their first time being split apart in different classes when the school transitioned from being a K-5 neighborhood school to being all 4th and 5th graders in their school system. These facts seemed to influence the children's design activities for the day, leading to several students creating designs as symbols of friendship – naming quilts in honor of classmates as a tribute to their friendships.

Like everything else in their school day, their time using DigiQuilt was different. For the last day, I did not provide the students with any particular challenges. They were free to make whatever they wanted. This freedom seems to correlate with a decreased level of mathematical discussion, but it is hard to say if that decrease can be attributed to the lack of particular challenges, the shortened time dedicated to DigiQuilt use, or to a general anticipation of the summer break. Perhaps it would be more surprising if any math talk occurred on this day (it turns out that the students did not discuss symmetry or fractions even once, though some of them created symmetric designs). With complete freedom, the students made a wide variety of designs. They also spent a fair amount of time looking at designs made by other children and altering them (like Talisha did with Peter's design in the following example (see Figure 32), also see Figure 31).

Talisha "messes up" Peter's Design

> Talisha is excited to try to change a design Peter made
> (3:22) Peter: "Can we make anything we want to this time?"

Me: “Yes, today you can make anything you want.”
Talisha: "Oh yes! I'm going for ninety-nine."
Peter: “What do you mean?”
Talisha: “I’m gonna make yours.”
Peter: “Which one? [pauses] Oh, that one. I’ll open it up for you.”

(4-5) Talisha tries to find the design that Peter made. She asks him if it is there. He says they’re in alphabetical order. Talisha has a tough time finding the design of Peter’s that she wants to change. She eventually does find it on her own.

After she is done changing Peter’s design, Talisha gets Peter’s attention.

(8:07) Talisha tells Peter to look at her design and he says, "Oh, you made that out of mine?" Joanna gets up at looks at Talisha's screen. Talisha looks at Peter’s screen. Peter says something I can’t understand that sounds like, “it’s *kind of* neat.” (See Figure 32.)
Peter (to Joanna): "She made that out of one of my design. She really messed it up!"
Peter (to Talisha): “You really messed it up.”
Talisha: “Go on now.” (She is chuckling. The students have been joking about “messing up” other people’s designs for the past couple of times using DigiQuilt, so this seems to be all in good fun.)

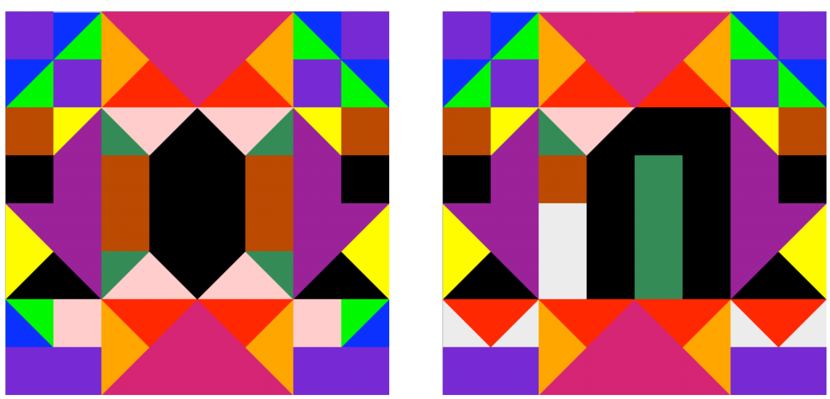
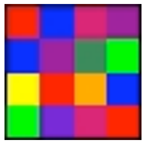
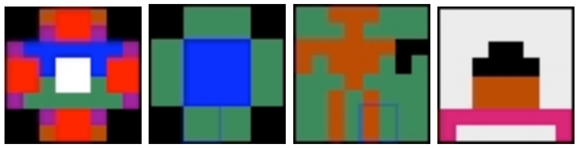
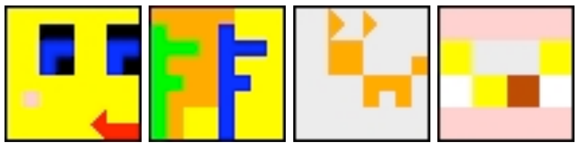
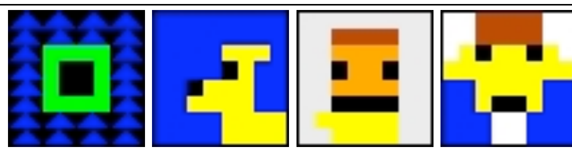
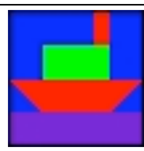
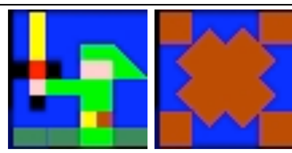
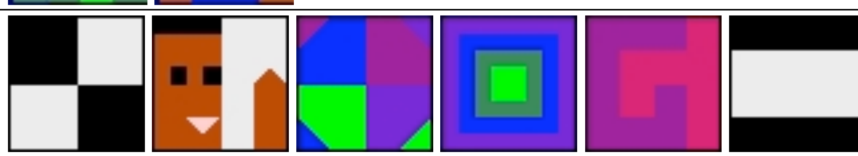
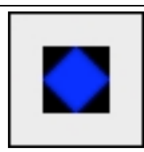
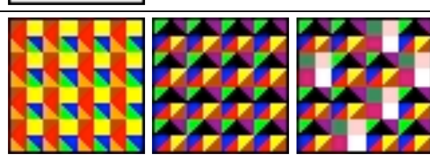
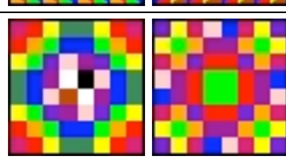
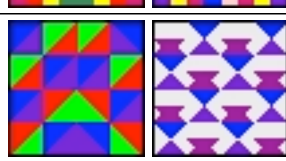


Figure 32. Peter’s design (left) & Talisha’s altered design (right)

The students only had 30 minutes for using DigiQuilt before they needed to take their post-test. During that time, the students created (or altered) 54 designs (see Table 9). That means that students each made an average of 2.8 designs, for an average time of around 11 minutes per design.

Table 9. Quilt block designs from day 7

Asha	
Austin	
Beth	
Carter	
Douglas	
Edzier	
Emma	
Imani	
Jake	
Joanna	
Keyla	

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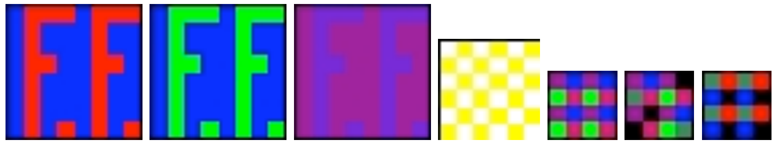
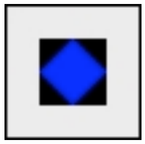
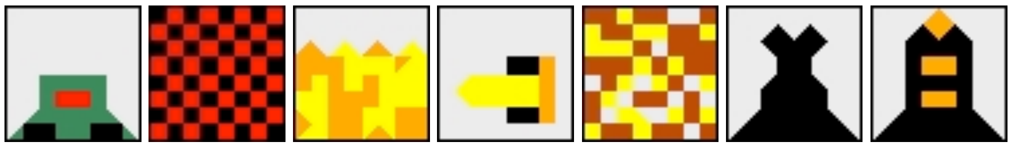
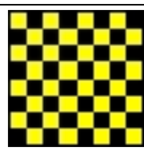
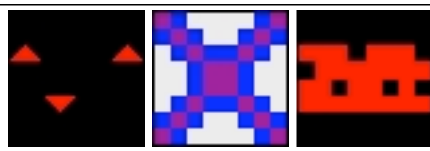
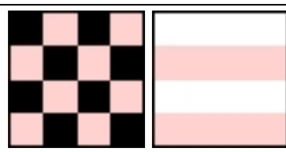
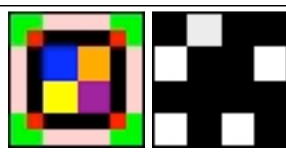
Lisa	
Omar	
Peter	
Ramona	
Serena	
Talisha	
Tyrel	
Wendy	

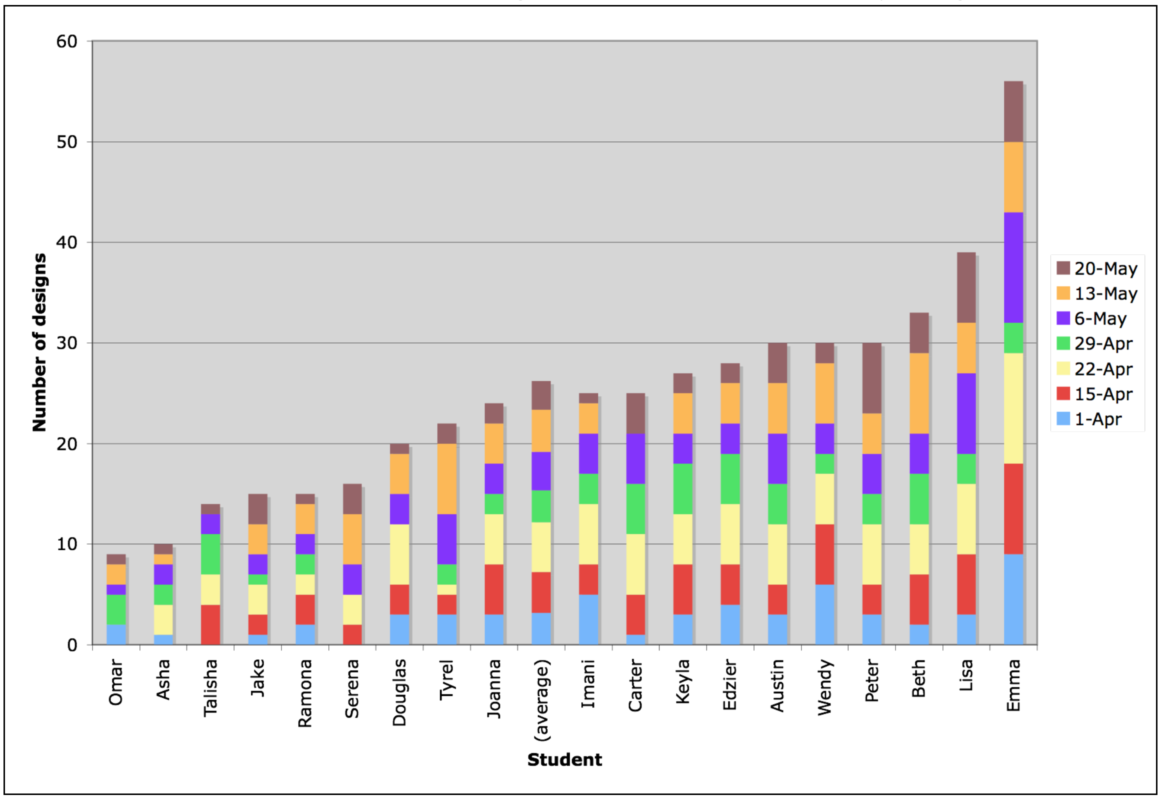
Summary of student design activities

Over the course of 7 class periods using DigiQuilt, students in this class created 468 quilt blocks. They designed the quilt blocks to solve challenges or just for fun. Along the way, they discussed their design activities with their friends, peers, teachers, and with me. They used the tools in the software as well as social structures to support their learning. They made goals that went above and beyond the work required to solve the challenges.

Though they were not always successful in reaching their goals, they often persisted through difficulty and were proud of the results.

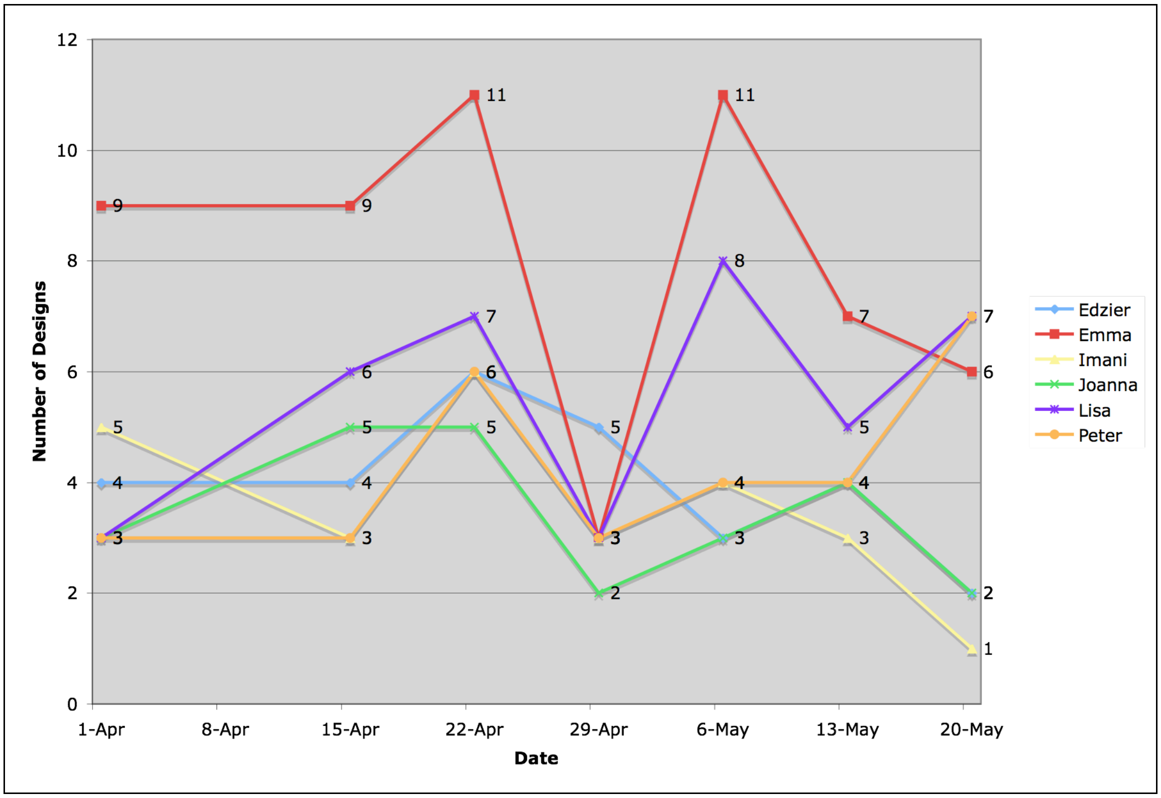
Some students created many more designs than did others (see Table 10). Emma, for example, was the most prolific designer, averaging 8 designs per day. She was the only student who ever created more than 8 quilt block designs in any given session (on two separate days, she made 11 designs each). Two students averaged fewer than 2 designs per day (their averages were 1.8 and 1.7). The average number of designs created per student per day was 3.7. That means that it took around 10 minutes on average to make a design.

Table 10. Total number of student designs as stacked totals for each day of DigiQuilt use.



There were some interesting patterns in the students' participation. The students I focused on generally created and saved a higher number of designs than the average student. It is possible that this is partly due to the fact that they were being observed. From day to day, students made varying numbers of designs. For one student, Emma, the difference in numbers for one day was particularly striking and coincided with spending a significant amount of time working on one difficult challenge (see Table 11).

Table 11. Number of designs made on each day by focus students.



The goal of this chapter was to show what happened over the course of one classroom's seven days of DigiQuilt use – to show what it looked like to use DigiQuilt in the classroom and give some specific examples of things that happened. In the next chapter, I will describe how I went about analyzing the data from this study.

CHAPTER 6

THE DESIGN OF THE ANALYSIS:

HOW I LOOKED AT THE DATA TO INTERPRET RESULTS

This chapter presents an explanation of my methods for analyzing the data from my dissertation study. The GW4 dataset was the cleanest dataset of the 4 classes. I used this dataset to lay out the range of possibilities. In addition to the pre- and post-DigiQuilt use data, DigiQuilt designs, and software log files, the GW4 dataset included more than 550 minutes of video data and completed challenge sheets from each child from each day (some more complete than others). I have reorganized, recombined, and looked at the data in many different ways in order to inform my hypothesis. My main focus has been the video data, relying on the other data as supplemental to the video to tell me more about the context or provide descriptive statistics rather than using it to establish any kind of statistical significance.

The summer after the data was collected, an undergraduate student assisted me with an initial pass on this set of video data to get a general feel for what was happening and noting times when especially interesting things were happening (interesting in terms of my hypothesis) – the video segments that were noted as especially interesting were then transcribed in more detail. By the time the data had been collected, I knew that I was interested in finding out ways that working within this socio-technical system might help children begin to mathematize their worlds.

Preparing to Analyze Video Data

Before data analysis could begin, then, I needed to specify what kinds of things I thought might be important to note. I knew I would not transcribe every detail of every minute of video data – there was way too much, and I was not looking for that level of detail for every minute of DigiQuilt use. Therefore, I decided I was particularly interested in times when children were talking about math. In addition, I was still interested in knowing what it looked like when children were engaged in creating designs they cared about, that reflected something about themselves, or that they seemed to identify with most.

I advised the undergraduate student who assisted me to watch the video looking for times when children were talking about math (making epistemological connections) and times when the students seemed particularly engaged or excited about their designs (making personal connections). One major clue, we decided, for determining if a child was engaged or excited, would be that they chose to share their design with someone. I indicated to the undergraduate student that I was still interested in the children's engagement over time and in the moment, so I was particularly interested in behaviors that indicated interest in the challenges, activities, designs, or math. I specifically wanted to know about times when the children stated opinions or thoughts about what they were doing or times when they were having conversations about their plans or accomplishments.

I had the undergraduate student look at the GW4 dataset because it was the cleanest dataset. In the meantime, I looked at the other video data with similar goals in mind – first, simply noting times that children were talking about math, then re-watching those sections to get more detailed information about what was happening, and finally writing in some transcription for those portions of the video. In sorting through and re-watching the GW4 video data, this initial "transcription" helped me go through the video more quickly while I was still fleshing out the transcription details for some of the most compelling examples.

The behaviors I looked for initially

Since this study was designed initially to study children's engagement with a constructionist design tool for craft and math, I knew that I cared about noting times when children seemed especially engaged with either creating designs they cared about or discussing the mathematical aspects of the challenges they were completing or the quilts they were creating. In other words, I wanted to note times when the children seemed to be noticing personal and epistemological connections. At this point, I knew more about what the video data was telling me, so I decided that it made sense to watch the video looking for times when students were doing the following things:

- Asking questions of peers about challenge
- Asking questions about a design
- Explaining a design
- Explaining a challenge
- Talking about math (to camera, teacher, or peer)
- Relating a design to a challenge
- Relating a challenge to a design
- Relating a design to a life experience
- Relating a life experience to a design
- Deciding which designs to share
- Sharing designs
- Complimenting someone's designs
- Asking about a design someone else made
- Asking a math question about a design someone else made
- Attempting to describe the math of a design
- Attempting to translate a design from one medium to another
- Attempting to create a design based on reality (a real life experience)
- Talking about designs that they have made in the past

- Remembering a design from another day (what do they remember about a design they made previously (without looking, which designs do they remember making)?)
- Remembering creating a design (what do they remember about creating a design when they look at it?)

All of these behaviors seemed to be the right kinds of things to watch for since I was interested in knowing what it looked like for a child to engage with a *constructionist* design tool for craft and math. I needed to know when they were *intellectually* engaged, so I needed to know what they were trying to accomplish and their understandings of the challenges before them. In addition, I needed to know when they were *emotionally* engaged, so I needed to know when they were particularly excited, frustrated, confused, or proud. Since the challenges were designed while keeping in mind some probable difficulties children were likely to have, and I knew what challenges they were working on, I could watch to see if the children were encountering difficulties when I anticipated they would and how they dealt with those difficulties. I knew what things I *thought* they would find most challenging, so I knew some of the times they might rely on support from the socio-technical system.

I looked for (and noted in the transcripts) times in the video data where these behaviors were especially obvious or common, so I could analyze those times more carefully. In this phase, I was trying to decide which cues to watch for, as well as watching for these listed behaviors.

I hoped that some patterns would emerge from the data. In particular, I hoped I would be able to say a lot about the patterns of behavior that seemed connected – which behaviors led to other specific behaviors. For instance, I thought there might be times when a child asked a certain kind of question, and following that question, I could predict what would happen next because there was a strong pattern in the data that would help me make that prediction. Or, perhaps there would be times that a child would say something (either to a peer, the camera, or a teacher), and what they said would matter in some specific way. I hoped I would see predictable activity patterns when a child worked on something in particular or transitioned between activities (I wanted to know what those transitions led to… learning, on task behavior, off task behavior, or whatever, so that I could say something about what behaviors might be indicated by certain other behaviors). Finally, I thought there would be times when a child was using a tool that was designed with a purpose in mind (maybe in the way I anticipated, maybe in another way or for another reason), and I would be able to say how the use of those tools related to the other kinds of cues that learning was happening.

Getting to a point where there were noticeable trends was more difficult than I anticipated, and the trends did not emerge as clearly as I had hoped. However, the kinds of trends I had hoped to find did inform my analysis methods. Thinking about the trends I thought I might see with respect to student behaviors like question asking, math talk, and tool use helped me focus on indicators that could inform me about those types of trends. I knew I wanted to look for and note I especially wanted to note:

- times when I heard children posing math questions or answering math questions,
- what the children had to say about their designs (particularly about the math of their designs), and
- how children interacted with each other and the tools in the socio-technical system.

Although the trends I anticipated I would be able to see were not as immediately obvious as I thought they might be, thinking about the trends I thought I might see helped steer my next steps in understanding this data.

Refining the way I looked for data that informed my hypothesis

After this pass on the data, I refined my approach to analyzing the data in terms of my hypothesis. I moved from a fairly long list of questions about engagement to a method of analyzing the hypothesis that split my efforts into looking for evidence related to predictions I had made about the two parts of the hypothesis:

1. **The "design" part of the hypothesis:** The children will be engaged with the design manipulative. They will enjoy using the software to express themselves. They will be excited about sharing their designs with family and friends. They will persist through difficult moments.
2. **The "bridging" part of the hypothesis:** The children will connect the concrete quilt block designs they create with the abstract "school math." By helping them view these artifacts in a mathematical way or interacting with these artifacts using tools that help them reflect on the artifacts in a mathematical way, we can help children mathematize their worlds.

This study was designed initially to study engagement. The story that seemed to emerge *related* to engagement, but seemed better suited to *describing* the engagement rather than tracking it. Both of these parts of my hypothesis relate to the larger idea of helping children do two things: 1) bring math into their worlds and, 2) notice math in everyday things. These relate to a larger goal of my research: help children mathematize their worlds so that they can experience math in everything they do and see. Mathematizing is all about seeing math in the world, and this study is related to that over-arching goal.

Both parts of my hypothesis relate to the idea of mathematizing. The first part of the hypothesis deals with getting children excited about mathematical activity (through design/art/craft activities) and trying to engage with art in a mathematical way. When the children in this study struggled to make their quilts look a certain way *and* solve a challenge, they were making their task artificially difficult. The act of adding extra criteria to the challenge for themselves, of adding additional goals, is a good indication of some level of adoption of this activity as interesting and worthwhile. When children persisted through difficulties they experienced, especially if those difficulties were the result of personal goals, that is definitely worth noting. If that happened (and it did), it would indicate that a child had set a personal goal above and beyond what was asked of them. Juggling extra design or artistic challenges on top of the mathematical challenges set forth might not have made the task easier (in fact, by definition, it made it harder), but for some students it seemed to add to the fun. Adding extra challenges would be one example of an indication of engagement with the activity on a more personal level, and a willingness to engage with the math in the face of added difficulty. I call this the "design" part of the hypothesis since it deals mostly with the idea that the children will be motivated to engage in the design process.

The second part of the hypothesis, which basically states that children are going to be able to make connections between abstract ideas and their concrete quilt blocks, is directly about mathematizing – about seeing math and talking about the math of examples in an attempt to learn more about the targeted concepts (fractions and symmetry in this case). The "bridging" part of the hypothesis doesn't stop at saying that the children will make connections. It further states that *we will help children make these connections* by helping them view their artifacts in a mathematical way, and by allowing them to interact with the artifacts using tools that help them reflect on their artifacts in a mathematical way. To describe the bridging that occurred, then, I would need to notice times when children were talking about math, what they were doing that led to or supported that math talk, and what that math talk looked like.

Once my approach to looking for data to inform my hypothesis was refined, I knew more about exactly what I hoped to be able to talk about using the data I had collected. I needed my data to inform these two parts of my hypothesis. Because I needed to be systematic about how I looked at the data, I decided I should tag all of the data in terms of things related to these two parts of my hypothesis so I would be able to look at the overall trends in the data in a more objective manner. The tags would allow me to search for trends, group the data in meaningful ways, and possibly spot patterns that were not obvious in the raw data. Of course, some features of the notes I took already allowed for rudimentary searches by things like date and student name. I needed to use tags as indexes to allow for more sophisticated searches.

First pass at "tagging" video data

On my first pass looking at the video data with this more refined approach in mind, I looked at the video specifically in terms of engaging with the design aspect of DigiQuilt and mathematizing. I noted times when learners were talking about their designs (sharing or discussing them in different ways, even if they weren't mathematizing). In addition, I noted all times where the students seemed to be mathematizing and whether the mathematizing was about a targeted concept (fractions or symmetry). I tagged approximately 4 hours worth of videos in this manner (see Table 12) before refining the tagging scheme and looking at all of the GW4 video data with a revised tagging scheme.

Table 12. Initial tagging scheme arranged by major themes

Challenges Reading/commenting on a challenge Showing how a design solves a challenge	**Planning/persisting** Planning a design Planning a design (describing a design idea) Using a tool in DigiQuilt to assist with planning a design Using tools to aid in construction of a design Persisting with a plan, in spite of difficulties Focusing on working on a design, even in the face of distraction
Making/finding meaning in the designs Naming quilts something from the world/finding real world things that look like a design Making a quilt block that looks like something from the real world Discussion that indicates some kind of sense of ownership/personal meaning Discussing a strategy for working with DigiQuilt	
Sharing/browsing Sharing a design Describing solution in terms of targeted math concept – symmetry Describing solution in terms of targeted math concept – fractions Using a tool in DigiQuilt to assist in sharing/explaining Looking at previous designs/sharing about designs Sharing about targeted math concepts – symmetry Sharing about targeted math concepts – fractions Wanting to see what someone else has done/checking someone else's work	**Talking about/seeing/understanding targeted concepts** Talking about targeted math concepts – symmetry Talking about targeted math concepts – fractions Asking/helping about targeted math concepts – symmetry Asking/helping about targeted math concepts – fractions Asking/helping about challenges Difficulty with a targeted content area - symmetry Difficulty with a targeted content area - fractions Using a tool in DigiQuilt to assist with targeted concept – symmetry Using a tool in DigiQuilt to assist with targeted concept – fractions Trying to figure something out using someone else's design

In the process of applying tags, I found that there were other episodes I wanted to note that did not fit very well with these choices. I had done a little too much predicting (and included predictions in the tags themselves) about what the mathematizing would *look* like – what kinds of effort would be directed at what kinds of activities. It was not my intent to only look for and tag episodes that fit predictably into these categories. It became clear that I needed to separate two ideas: the kinds of effort and where the effort was directed, so that I could more easily tag all of the engagement with design activities or mathematizing or apparent attempts at mathematizing that were present in the data.

I initially imagined that I would watch the video and record what kinds of mathematizing or other engagement were happening in any given minute. This kind of approach seems to be common in studying engagement – each unit of time is tagged with what was going on and then tallies are used to describe what happened overall or how much time was spent doing a certain thing. This kind of coding seemed inappropriate for this set of video data. Though I had predictions about what I might see, I did not have a small enough list of things to look for that this tallying method would be useful. Also, I did not have enough video data about any one child to track their engagement at that level of granularity.

Refining the video data tagging scheme

In order to listen effectively to what the data was telling me, I needed to look at it in a more organic way that involved less processing of moments and statistical analysis and more clumping of moments into describable groups or "episodes," and looking for trends amongst the episodes.

In developing a set of tags that related to "what the participant was doing" and "what the doing was about," I created a chart with the "doing" and the "what the doing was about" as the axes. I filled in examples of what most of the pairs of tags might look like. Eventually, I decided that might be too constraining, so I left the examples there as a guide, but tagged the actual video trying to maintain an open mind about times to apply various tags. I broke the table into manageable-sized pieces and drew (literally drew) some connections between the parts of my hypothesis and the parts of the table. I wrote short descriptions about how I thought different parts of the table informed my hypothesis.

The entire table is too large to present at one time in this style of document, so I split the table into groups according to "what the doing was about." The parts in bold represent the actual tags (the headers of the columns and rows), and the boxes where columns and rows match up include examples of what an episode might look like if it would get that combination of tags.

The first portion of the tag table (see Table 13) includes all the different kinds of "doing" that are about designs (rather than targeted content areas). This portion of the table includes most of the things that would indicate engagement with the parts of the activities that would relate to being part of or designing for an audience. Unfortunately, it is much more difficult to determine when a child is doing some of the introspective things in the bottom few rows, so when applying those tags, I relied on what they are saying or other cues such as gestures and facial expressions. Even then, I could only make informed guesses.

Table 13. All the design tags (in bold) and examples of combinations of the tags

What is the -> doing about?->	**Design**		
What is the subject doing? \/ \/ \/ \/ \/	**A design**	**A design idea or plan**	**How a design solves a challenge**
Sharing/ Announcing/ telling/ commenting/ checking/ asserting	Sharing a design	Stating a design idea or plan	Stating that a design solves a particular challenge
	Saying something about a design/ Complimenting a design		
Describing/ discussing	Discussing a design	Discussing a design idea or plan (more collaborative)	Describing ways a design solves (or not) a challenge
Asking/Helping		Asking for design ideas to improve a design	Asking for help determining if a design solves a challenge
Watching/ looking/ browsing	Looking at/ browsing through previous designs		
Using a tool in DigiQuilt	Using a tool in DigiQuilt to make a design	Using a tool in DigiQuilt to plan/ inspire/ layout a design	Using a tool in DigiQuilt to show how a design solves a challenge
Making Meaning	Naming a design after something in the world		
Figuring out/ building/ developing understanding		Making a quilt block that looks like something in the real world	Figuring out how a design solves a challenge
Experiencing Difficulty	Difficulty trying to make a design look a certain way	Difficulty making a design as planned	Difficulty describing how a design solves a challenge
Persisting	Persisting in spite of challenges or difficulties experienced while attempting a design		

The next two tables are about fractions and symmetry. The sets of tags for these two areas are nearly identical. These top portions of each of these tables (see Tables 14 and 15 up to the double line) contain the tags related to talking about fractions or symmetry – the two targeted-content areas. The leftmost column describes the subjective parts of the label – exactly how I determined what type of "doing" was happening and the judgments I was making. The actual tags are the parts in bold. In each place where a pairing of tags meets up, I have written a description of a possible episode that would receive that pair of tags. Sub-columns such as "symmetry of a design" and "symmetry in general" indicated additional modifiers that would possibly be included if they fit. This part of the table deals most directly with the bridging part of the hypothesis, but would also include episodes where children were engaging with math in any way (whether or not bridging was involved). Episodes tagged with the extra modifiers "of a design" would be likely to be included in instances that involved bridging – they would be examples that help us show that the designs created using DigiQuilt actually do afford "math talk" about them.

The portions of these tables that follow the double lines deal with the same "what the doing was about" columns, but move further down into other kinds of "doing." This portion of the table of tags and examples includes tags that would indicate times when the software tools that were provided seemed to play a role in helping children view their artifacts in a mathematical way or helping them reflect on their artifacts in a mathematical way. This set of tags also includes some types of "doing" that are difficult to identify – especially making meaning and figuring out/ building/ developing understanding. These are things that show that kids can and do use the quilt blocks as things to think with – they can use their designs to explore and experiment with targeted content ideas. While the first tags dealt with talking about math, these deal with seeing math. As in the previous table, some things are difficult to observe, so those tags would only be used if there were some indirect indicators "that kind of doing" was happening.

Table 14. All the fractions tags (in bold) and examples of combinations of the tags

	What is the -> doing about?->	**Fractions**	
	What is the subject doing? \/ \/ \/ \/ \/	**Fractions of a design**	**Fractions in general**
more or completely one sided/ initiating a discussion/ getting someone's attention/ requesting or being audience	**Sharing/ Announcing/ Telling/ Commenting/ Checking/ Asserting**	Saying that a design has a certain fraction in it	Sharing knowledge about fractions in general
		Commenting on / Complimenting someone's skills in a targeted concept	
engaged conversation/ exchange of information	**Describing/ Discussing**	Describing a design in terms of fractions	Discussion of fractions
similar to describing/ discussing, but usually carried out in terms of one person's questions	**Asking/Helping**	Asking/ helping about the fractions of a design	Asking/ helping about fractions in general
Might indicate seeing or seeking math in the quilts	**Watching/ looking/ browsing**	Watching someone else figure out something fractions related	
Some of these could probably really just be indicated as "with tool" in other spots if separating out verb from object/subject does not matter	**Using a tool in DigiQuilt**	Using a tool in DigiQuilt to assist with making a design have a fraction or showing a fraction in a design	Using a tool in DigiQuilt to explore or figure out a fraction or share or explain something about fractions
Discovering or "seeing" math	**Making Meaning**	Coming up with a way or adopting a way of describing the fractions of a design	Coming up with a way or adopting a way of describing fractions
	Figuring out/ building/ developing understanding	Figuring out if a design has a particular fraction	Figuring out a fraction / playing with fractions / building an understanding of fractions
These two rows are intimately linked – their tie is meaningful. Persisting implies that there was something difficult.	**Experiencing Difficulty**	Experiencing difficulty with the fractions in this design	Experiencing difficulty or confusion about fractions
	Persisting	Persisting in spite of challenges or difficulties experienced while attempting a design	

Table 15. All the symmetry tags (in bold) and examples of combinations of the tags

	What is the -> doing about?->	**Symmetry**	
	What is the subject doing? \/ \/ \/ \/ \/	**Symmetry of a design**	**Symmetry in general**
more or completely one sided/ initiating a discussion/ getting someone's attention/ requesting or being audience	**Sharing/ Announcing/ Telling/ Commenting/ Checking/ Asserting**	Saying that a design has some kind of symmetry	Sharing knowledge about symmetry in general
		Commenting on / Complimenting someone's skills in a targeted concept	
engaged conversation/ exchange of information	**Describing/ Discussing**	Describing the symmetry of a design	Discussion of symmetry
similar to describing/ discussing, but usually carried out in terms of one person's questions	**Asking/Helping**	Asking/ helping about the symmetry of a design	Asking/ helping about symmetry in general
Might indicate seeing or seeking math in the quilts	**Watching/ looking/ browsing**	Watching someone else figure out something symmetry related	
Some of these could probably really just be indicated as "with tool" in other spots if separating out verb from object/subject does not matter	**Using a tool in DigiQuilt**	Using a tool in DigiQuilt to assist with making a design symmetric or showing symmetry in a design	Using a tool in DigiQuilt to explore symmetry or share or explain something about symmetry
Discovering or "seeing" math	**Making Meaning**	Coming up with a way or adopting a way of describing how a design is symmetric	Coming up with a way or adopting a way of describing symmetry
	Figuring out/ building/ developing understanding	Figuring out if a design is symmetric	Figuring out symmetry / playing with symmetry / building an understanding of symmetry
These two rows are intimately linked – their tie is meaningful. Persisting implies that there was something difficult.	**Experiencing Difficulty**	Experiencing difficulty with symmetry in this design	Experiencing difficulty with symmetry
	Persisting	Shows a commitment/level of engagement that says they care at least enough to stick with it in spite of difficulties – this might show some level of personal meaning.	

The last portion of the table of tags (see Table 16) deals mostly with the patterns of progress that the children developed. These tags indicate times when children's activities were impacted by their peers, the challenge sheets, the software, or some combination of those. These tags deal mostly with the logistics of the socio-technical system. For more introspective activities, such as browsing through previous designs, if there was not any talking or other indication, it was difficult to know if the child was making or looking for any connections between their actions and the task at hand. To some degree, these activities do tell us about what motivated children. The tags about the challenges tell us something about the role the challenges played in the socio-technical system.

Table 16. Tags relating to parts of the socio-technical system (in bold) and examples of combinations of the tags

What is the -> doing about?->	**Progress/work accomplished**		**DigiQuilt**	**A Challenge from the sheet**
What is the subject doing? \/ \/ \/ \/ \/	**Self-progress**	**Others-progress**		
Sharing/ Announcing/ telling/ commenting/ checking/ asserting	Telling others how far you have gotten	Asking others how far they have gotten	Revealing/ Telling a strategy for working with DigiQuilt	Reading aloud or commenting on a challenge
Describing/ discussing			Discussing a strategy for working with DigiQuilt	
Asking/Helping				Asking/ helping about challenges
Watching/ looking/ browsing	Looking at own previous designs	Looking at previous designs of others	Watching a demonstration of some technique for using DigiQuilt	Reading through challenges
Using a tool in DigiQuilt	Using the browsing capability in DigiQuilt to look back at own work	Using the browsing capability in DigiQuilt to look back at others' work	Using a tool in DigiQuilt for the first time/ exploring DigiQuilt features/ playing with tools	
Experiencing Difficulty	Distraction - announcement, someone trying to get attention, other things going on		Difficulty making DigiQuilt do what is desired	Difficulty in solving or understanding a challenge

Tagging the video data with the refined coding scheme

In this more refined pass at tagging the data, I gave each "episode" one or more combinations of tags that described what the participant was "doing" and "what the doing was about". The episodes were sometimes momentary, and sometimes lasted close to 20 minutes with smaller sub-episodes included within them. I use the word "episode" rather than "moment" because the instances that felt appropriate for tagging varied greatly in length. Though some episodes were short, others were long and rich examples filled with many mini-moments of mathematizing. In terms of what happened on any given day, I considered an episode to be any series of related events that seemed to go together in such a way that separating them deprived the story of its richness. For example, if a student started out struggling and I have captured on video several events or moments that combine to tell the whole story about how he eventually solved the problem, that was an episode. There were 312 episodes in the GW4 data (not counting the sub-episodes or mini-moments that made up the episode).

This system of breaking up the data allowed me to describe the kinds of things I was seeing and how frequently I was seeing them. It does not allow me to say anything about the proportional amount of time that was spent engaging with the socio-technical system in any particular way, but my goal is to describe what kinds of things are possible rather than how much time is spent in a particular mode. There were not any episodes that were deemed interesting enough for looking at more carefully for which I could not find any appropriate tags, and these tags seemed to cover the range of things that I needed to be able to describe or keep track of.

Most episodes (and sub-episodes) were tagged with at least 2 combinations of tags. For example, an item tagged with [Talking math][Symmetry], might also be tagged with [DigiQuilt Tool (DQT)][grid][Symmetry][of a design] where I would actually use the tag

[DQT] to indicate the use of a DigiQuilt tool rather than writing it out, and [grid] specifies that it was the select-a-grid tool. [Symmetry] is what the doing was about, and [of a design] simply indicates that the symmetry was related to a specific design (which was most often the case).

My analysis of the data

Once all the episodes were assigned tags using the tagging scheme I have described, I worked on analyzing the data in terms of the two parts of my hypothesis. For each part of my hypothesis, I looked at the data by counting the number of times certain tags were applied or grouping episodes that related to that part of the hypothesis. Looking at the data in these two different ways informed my hypothesis in different ways. Counting the number of times a particular tag was applied (or the context in which it was applied) gave me some idea about how common it was for certain things to occur. This was particularly true for searches on combinations of tags, since the same combination is unlikely to be used for the same episode (so the count tells how many episodes had that combination of tags). Using the tags to create groups of episodes to look at more closely, I used each search to help me expand the group of episodes. That means that my focus shifted, in the moment of the search, from looking at how the tag was applied or how frequently it was applied to looking for ways to find more episodes that I had not already included in the group. The focus, then, is on growing the dataset rather than describing the dataset (at least in the moment that the search is happening). For the engagement part of the hypothesis, I focused more on the former style of looking at the data, and for the bridging part of the hypothesis, I used the latter approach. In both cases, the tags allowed me to search my data in a meaningful way and supported the analysis that followed.

Analyzing the data in terms of the "design" predictions

As stated earlier, one part of the hypothesis of this study was that children would engage with the design approach. The tags I chose to use to index the video data included some that were related to the design aspects of what the children were doing. I wanted to be able to show evidence related to the children's engagement. I imagined that I would need to show several things:

- that the children cared enough about their designs to share them with others
- that the children would persist through difficulties – they would want to make their designs look a certain way, and they would be wiling to struggle to make that happen
- that the children would incorporate their everyday lives into their designs or create things from the real world

The types of evidence I need to show to inform this part of the hypothesis is much easier to spot than bridging. In order to be able to talk about the children's engagement with the design aspects of their DigiQuilt use, I first looked to the tagged video data to find some quantitative information. Just counting tags, I was amazed how frequently the tag [Design] was applied. Just sheer numbers of times the tags were applied (might be more than once per episode), compared to [Symmetry] which was used 116 times and [Fractions] which showed up 44 times in the tags for the 312 episodes, [Design] was used 327 times! Since that tag differs from the tag [of a design], this 327 does not include the 55 times that the [of a design] tag was used. Every time the [of a design] tag was used, it was used for either [Symmetry][of a design] (37 times) or [Fractions][of a design] (18 times). So, when the "of a design" tag was applied, it was likely in the context of some mathematizing, while time that the "design" tag was applied, the design was the main thing that the doing was about.

I wanted to show that the children cared enough about their designs to share them with others. I looked to my tagged video data to find and count instances where sharing was happening. The tag that I used when the children were sharing anything was [Sharing/Telling/Announcing/Commenting/Checking/Asserting (STACCA)]. When it was especially clear that the kind of action that was happening was "sharing" as opposed to some other verb from the list in the [STACCA] tag, I also included the tag [Sharing] in the sequence of tags. So, the combination of tags [Sharing][STACCA][Design] would indicate times when a child was sharing a design. Of the 99 times that combination of tags occurs in the 312 episodes, there were only a handful of times when a child was sharing a design made by someone else. Sheer numbers would suggest that sharing designs with others was a popular activity of the children. I will show examples of this sharing in the next chapter.

I wanted to show that the children would persist in spite of difficulties because that would show a level of engagement that suggests that the children care about meeting their design goals. I searched for the tag [PER], which stands for "persisting," to group the episodes that included times when children noticeably persisted through difficulty. I looked at these episodes more closely to try to understand more about why a child kept working in spite of difficulties – what was the motivating factor?

Finally, I wanted to find times when children were connecting designs to things in the real world. The combination of tags I searched for to find those times: [Design][how a design connects to something in the world] occurred 66 times. I grouped episodes tagged this way in order to try to spot trends.

- 19 times, the connection seemed to be made through naming,
- 20 times, the children seemed to be making or telling a story,

- 14 times, the children were describing or discussing how a design connected to something in the world,
- 10 times, the children seemed to be making meaning, finding out, building or developing an understanding,
- 3 times, planning a design or thinking about how it could be created in a way that would make it like something in the real world.

I looked at these episodes and the designs related to the episodes to find out more about what seemed to be exciting for the children. I was especially looking to see not only how they seemed to feel about their own designs that resembled things from the world, but also how they reacted to designs made by other children that attempted to do the same.

Analyzing the video data in terms of the "bridging" predictions

I was hopeful that looking at my tagged data would allow me to categorize the different kinds of interactions that led to bridging. I thought the best way to do that would be to pull together all the episodes that related to fractions according to what was happening with the fractions (and the same for symmetry). I thought this would help me because it seemed likely that certain kinds of actions (e.g., describing and discussing, asking and helping) would be more likely to lead to bridging than others, and that pulling together the episodes that were tagged in ways I thought would most likely result in bridging first was the easiest way to organize things. I imagined that once the episodes were grouped in this way, trends would emerge suggesting what kinds of math talk or explorations were most likely to lead to bridging.

Using the tags as indexes, I kept a log of the searches I did for each combination of tags, why I did each search, and what I found. I kept track of how each new search helped me learn more about fractions or symmetry bridging accomplished by the children. For each search, I used my indexing scheme (my tags) to pull together episodes for closer examination. With each new search, I added episodes that were not already included in whatever set of information I was gathering. My focus was on creating a set of episodes by doing a successively broadening series of searches.

<u>Grouping fractions episodes</u>

I began by searching for episodes that might include fractions bridging. With each new search, I added episodes that were not already included in the results of previous searches. Since I thought that describing or discussing suggested deeper engagement than any of the other actions related to fractions, I first found all the episodes where the kind of "doing" was describing or discussing and "the thing the doing was about" was fractions. This **added 5 episodes** to my set. Then, I added any episodes that were tagged with talking math and fractions. This time, only **4 episodes** were added. I recalled that this tag was a more general tag that I did not use very often because there were more specific tags that I could use instead. After looking for general math talk about fractions, I looked for the tag combinations that suggested that children were specifically dealing with the fractions of a particular design. This **added 14 episodes** to the set, and seemed likely to be fruitful for information relating to bridging since there would be a specific design involved with whatever was happening. Then, I wanted to add any episode that involved fractions in some way, but not in the ways previously described. This **added 7 episodes**.

Having collected 30 episodes that I thought would be likely to include fractions bridging based on the "kind of doing" that was happening, I wanted to add any episode that involved the tools that were in place specifically for supporting fractions learning, so I looked for episodes tagged with "fractions feedback" and then the combination of "grid" and "fractions." There were 8 episodes where fractions feedback was one of the tags (though closer inspection of these episodes led me to believe that fractions feedback was involved 13 times). There were 3 times that the grids were used for fractions (in an obvious enough way to be captured on video). A few of these episodes that involved DigiQuilt tools involved both tools, so that made for a total of 10 episodes where at least one tool was involved. All 10 of these episodes were already included in the 30 episodes previously listed, so this search did not help me grow the dataset, but was interesting to note that software tools seemed to play an obvious role in fractions bridging on 10 different occasions, and that the grids were only used for fractions 3 times out of the 30 fractions bridging episodes.

<u>Grouping symmetry episodes</u>

I went through a similar process for creating a grouping of episodes relating to symmetry. In this data search, I tried to find examples of children making connections between the concept of symmetry and the designs they are creating. First, I identified and added to my grouping of symmetry episodes those episodes that were tagged in a way that suggested that children were describing or discussing symmetry ([Designing/Discussing (DD)][Symmetry]) since those included discussions of the symmetry of a design, which seems like one important way that the children might make the connection. There were **14** examples of describing symmetry. In addition, I looked at times that [Talking math][Symmetry] occurred. There were **10** times when I tagged something [talking math][symmetry] (but not [DD][Symmetry]). Then, I looked for times in addition to those when the tag includes [symmetry][of a design] since those seem to include sharing, asserting, and announcing… if not describing and discussing. Since the students were still dealing with the designs, it seemed likely that these episodes would probably connect to the bridging idea. There were **11** times that “symmetry of a design” was tagged. That was the last search I did to look for additional episodes about symmetry. There were **3** episodes where children did [Sharing/Telling/Announcing/Commenting/Checking/

Asserting (STACCA)][Symmetry] in the context of their designs: one where the child seems to explain a strategy for making symmetric designs, and two where children are checking for symmetry (but one of those seems to be a request for verification rather than the child actually checking). Looking for times that children were having difficulty with symmetry yielded **2** additional episodes. That meant there were 40 symmetry episodes gathered so far (recall that there were 312 total episodes), involving 12 different students (out of 19 student participants, though not all students were videotaped as closely, so this distribution is more spread out that I anticipated).

From there, I found out how many times an episode or part of an episode involved using the grids to do something with symmetry by looking for [grid][Symmetry]. There were 8 episodes with that tag (some of these were repeats of episodes already found). There was one additional episode where the grid was used, seemingly in the context of symmetry, but the connection between the use of the grid and something symmetry related is less clear. So, of the 40 symmetry episodes gathered at that point, 9 of them involved the select-a-grid tool.

In addition to these 40 symmetry episodes, there were 7 times when children seemed to be experiencing difficulty with symmetry. Some episodes showed children struggling with symmetry and seeking help, but some of these episodes involved children not quite getting symmetry in their designs, and not necessarily figuring out that they needed help at all.

<u>Results of initial groupings of fractions and symmetry episodes</u>

My first pass at analyzing the data yielded some useful numbers, but the episodes as they were gathered in the groupings lacked the context and continuity that would allow me to reveal any trends. The first pass at the grouping and analyzing the tagged episodes suggested that I needed to find another way to look at the data that would allow me to describe more details of the rich and nuanced tales included in the data.

I believe the reason that my first attempt at working with the tagged data did not yield the results I expected it would was that I took each episode so far out of context that it was difficult to piece together what was happening. For instance, if two chronologically adjacent episodes dealt with symmetry, but in slightly different ways, those two episodes could be added to the grouping in such different places (possibly even out of order) that it would make it difficult to see how they related. The data was not sending a clear message when it was grouped in this way, and this made meaningful analysis difficult.

My first pass at this type of arrangement showed me that the students were supported in their bridging in a variety of ways – sometimes through the software, sometimes through social aspects of the socio-technical system, and still other times, there was not quite enough support for them to succeed at their bridging. This suggested that I needed to find a way for the data to tell me more about what supported the children's bridging and the context of the bridging. Because the first manner of organizing the data pulled the episodes out of chronological order and grouped them by the type of action, it was tricky to say much about the context of the bridging. I had the feeling that fractions and symmetry were different enough to keep them in separate groups, but I could not explain the difference and the structure I had thus far imposed on the data was not helping me make the case for separating them.

I felt that this limitation I experienced while analyzing the data was not stemming from the tags themselves, but from how I was using the tags. I needed to figure out a new way to look at the data that allowed me to easily follow along chronologically with the episodes, but still consider them as groups with similar tags.

Second pass at analyzing the tagged video data in terms of the bridging predictions

In order to be able to say something more meaningful about the data, I needed to arrange it in a way that let interesting patterns emerge, and I still needed to be systematic. My second approach made it easier to think about the episodes in their original context. I knew I needed to keep the episodes largely in order so that I would have an easy time fitting them into the full story when I needed to find out more about the surrounding activities and events. I also knew that I was more interested in how the episodes were similar or different from each other within and between each category than I was in keeping episodes with the exact same tags near each other.

I first pulled together any episode that was labeled "fractions" no matter what the other designators were. From there, I created a document that contained all of the fractions episodes, in order but separated from the rest of the episodes. I wrote short descriptions for each episode that described first, if I thought it seemed like the episode involved bridging, and second, if it did involve bridging, what seemed to be supporting the bridging. I did the same thing for symmetry (pulling episodes with symmetry tags, looking for bridging, and describing what seemed to support the bridging). This method of organization allowed me to describe the results more clearly because it highlighted and accentuated nuanced differences rather than grouping the episodes and masking some of these important differences.

Once I had created these two documents and written short descriptions about what seemed to be supporting any bridging the children were doing, I color-coded some of the tags according to the type of activity that was taking place. I wanted to see if there was any pattern as far as when a student might ask about something or assert something. Color-coding was not enough to make any patterns stand out.

Finally, I made maps of the bridging episodes for fractions and for symmetry that grouped like episodes together so I could detect any patterns that might emerge. I did not see any major patterns within either fractions or symmetry, but I did notice a contrast between the two that I had not noticed before. The math talk about fractions and symmetry was both qualitatively and quantitatively different.

In the next chapter, I present the results of this analysis.

CHAPTER 7

RESULTS

In my dissertation study, I explored what happens when children interact within a socio-technical system designed to support their math learning through the use of manipulatives for design (i.e., design/art/craft). The literature suggested that a design approach would be engaging for learners. The math learning literature suggested that manipulatives would be a productive way to help children connect the concrete and abstract. However, previous work with manipulatives seemed to focus either on supporting design activities *or* supporting children as they attempted to connect the abstract and the concrete. My hypothesis was that children would be able to leverage the affordances of both the design approach *and* the use of concrete materials for supporting "bridging" between the abstract and concrete if they had the right kinds of supports for doing so. Previous work on learning through design has suggested that, although the design approach is engaging, it can be difficult to help the learners connect their design experiences to the targeted content knowledge (e.g., in Learning By Design, helping the learners connect their design activities to the science content has been a major focus – it is easy for learners to get caught up in design and forget about making these connections). In this study, I explored the ways children engaged within the socio-technical system, looking in particular at their engagement with the design approach, their mathematizing, and their mathematical bridging in order to understand more about the supports that they seemed to need and use at various points in their learning in order to bridge between their designs and the targeted concepts – what seemed to help them connect the abstract and concrete.

In this chapter, I present the results of my analysis of the data from my dissertation study. I will describe the analysis of the data from two perspectives. First, I will describe trends that emerged in students' patterns of engagement according to my preliminary analysis of the tagged video data. I begin by highlighting some trends that seemed to emerge from the data. I will focus here on utilizing some descriptive statistics relating to the video analysis.

The other level of analysis will be from a bit higher level related to the overall story that unwrapped throughout the study. For this level of analysis, I will present the results of this study in a structure that echoes the predictions related to my hypothesis. I will start with the first part of the hypothesis: the students will be motivated and engaged – they will find something that is personally meaningful to them in this design activity. Students showed that they cared about their designs by sharing their designs often, connecting their designs to things in their world, and expressing the desire to share their designs in a variety of ways. They also became engaged with design as part of the audience – they sought opportunities to praise each other and ask about their peers' designs. They talked about the math in their designs. Then, I will present data that informs the second part of my hypothesis: the students will make connections between the concrete and abstract. Students succeeded in making and sharing connections between their quilt designs and the targeted concepts of fractions and symmetry. They did this without leaving the motivational benefits of the design context behind. This was possible in part because of the ways that bridging was supported within the socio-technical system.

For results of the analysis at both levels, I will pull examples from the other classrooms that seem relevant either because they are aligned with what happened in the GW4 classroom or because they offer the opportunity to highlight a nuanced difference between this data and data I saw elsewhere. My goal in choosing GW4 as the focus class was that I would be able to lay out the range of possibilities, but there were a few times when the GW4 examples were indicative of, but not as rich as, examples from other classrooms, or where data from other classrooms could be used to provide additional useful information.

Patterns of engagement: The numbers

As I mentioned in the previous chapter, on my second attempt to analyze the video data, I grouped the data into three sets – one each for design, fractions, and symmetry – by gathering episodes that included student actions related to students' engagement with design and the targeted content areas. I counted the total number of episodes where design was the central topic, and symmetry and fractions episodes where bridging seemed to occur (where there seemed to be some connection drawn between the targeted math concepts and the concrete quilt block designs). It occurred to me that there might be some interesting patterns at the daily level that would indicate how the challenges related to the amount of sharing or kinds of bridging supports that were needed and utilized by the students. Since there was no sound on the first day's video record, I do not know how many times the students seemed to engage with design, symmetry, and fractions. I counted the number of design, fractions, and symmetry episodes for each remaining day (see Table 17). Two interesting trends emerged: There were many more interactions related to the design aspects of the children's activities, and there were more interactions where the math topic was symmetry than fractions.

Table 17. Number of episodes related to design, fractions, and symmetry on any given day in GW4

	Design	**Fractions**	**Symmetry**
April 15 – Day 2	17	3	19 (25)*
April 22 – Day 3	46	6	16
April 29 – Day 4	34	6 (12)*	9 (13)*
May 6 – Day 5	29	7 (14)*	1 (7)*
May 13 – Day 6	22	2	3
May 20 – Day 7	36	0	0
Totals	**184**	**24 (37)***	**48 (64)***
* Numbers in parentheses indicate approximate number of related episodes if sub-episodes were also counted			

More interactions related to design than to targeted math content

Design was the topic of children's interactions more frequently than fractions and symmetry combined. Though on the second day, the gap between fractions and symmetry is greater than the gap between design and either type of math talk, the remaining five days suggest that design was the topic of discussion or sharing much more frequently than either fractions or symmetry. In fact, the number of times that fractions and symmetry were talked about combined still adds up to fewer episodes than those where designs were at the forefront. In the next chapter, I detail some of the possible reasons for this disparity.

One student, Edzier, seemed very excited to have the opportunity to create whatever quilt designs he wanted on the last day, saying, "This, ah, something I've always wanted to do." Interestingly enough, though, he did not create any designs from scratch and did not even alter the second design he opened using the block browser. Earlier, for example on days 2 and 3, there was definitely discussion amongst the students about who was "free-quilting" because they had finished their challenges. The students often paid close attention to each other's progress, asking each other which challenge they were on or bragging about how many designs they had already completed.

The complete lack of math talk on the seventh day surprised me a little bit, but not a lot. Even without assigned challenges, though, the children remained engaged with DigiQuilt. They set design goals for themselves and were proud of the things they made. They were also generous with praise for each other's accomplishments. They made personal mementos for their friends – not just printouts of their designs they made anyway, but designs that were both created and named for a particular friend in honor of their friendship. These designs were truly special to both the creator and the recipient.

More interactions relating to symmetry than fractions

The learners were visibly engaged with symmetry (discussions, gestures, working on designs, struggling with conceptual understandings) about twice as many times as they were visibly engaged with fractions. I will describe the qualitative differences between fractions and symmetry data later in this chapter, but this quantitative data was striking. Since these students were already working on fractions when this study began, but they had not heard very much about symmetry, it seems likely that they were struggling with symmetry in many ways. Some possible reasons for this quantitative disparity will be described in the discussion of these results.

Exploring children's engagement

The first part of my hypothesis was that children would engage with the design approach. Specifically, my predictions related to this part of the hypothesis are:

1. The children will be engaged with the design manipulative.
2. They will enjoy using the software to express themselves.
3. They will be excited about sharing their designs with family and friends.
4. They will persist through difficult moments.

There was evidence related to all four of these predictions.

How children engaged with the design manipulative

It was easy to see that children were excited to create designs in DigiQuilt. The GW4 class created 468 quilt block designs over the course of about 4.5 hours of DigiQuilt use. In classrooms where DigiQuilt was not a whole-class activity, many students chose to use DigiQuilt (even when other options included math games or free-play). They were eager to share their designs with their peers, teachers, friends, and family. In the GW4 class, designs were the topic of student interactions in 184 out of 312 episodes, and that does not include the 55 episodes where the symmetry or fractions of a design were the topic of discussion.

There were 23 times in the GW4 data that students remained engaged with DigiQuilt, even in spite of some distraction – whether it was Peter asking Lisa for help and Lisa asking him to hold on while she finished, or Emma asking Lisa to wait a second before she could take a break to look at a design, or many students not wanting to stop working while they listened to announcements or directions (especially at the end of DigiQuilt time for the day) – students were reluctant to stop using the software.

Six students from GW4 and several other students from the other classroom at GW brought a copy of the software home to use it during their free time. At least one pair of students got together outside of school and used DigiQuilt to make designs. One student's little brother made some designs that his dad printed on business cards for him. Students from CH Elementary used the software outside of class too – sharing the software with parents, siblings, caregivers, and friends. Some of these designs were saved on the laptop computers, so I was able to see them. One student told me about showing her leader at the Boys and Girls Club how to use DigiQuilt to make designs. She seemed proud of being able to share not only her designs, but also the software and the experience of creating designs with her leader. I created a sheet with challenges for children to use I their free time if they wished, and every student who had the software at home elected to take one of the challenge sheets.

How children used the software to express themselves

The children in this study definitely used the software to express themselves. Sometimes, they were not only creative in the design aspects of their quilt blocks, but also in the interpretation, storytelling, and naming that related to their designs. In the tagged data from the focus class, there were 20 times when naming designs was the topic of conversation – discussing the appropriateness or finding suggestions for names. There were 67 times that how a design connects to something in the world was the topic of student talk. There were 36 times where a child was making or telling a story about how their quilt design related to the world or sharing a design idea or plan about how a creation would eventually turn out. The students used the software to express themselves by:

- Creating designs that looked like things from their lives
- Creating designs that held special meaning or stories, and
- Naming designs in ways that suggested personal meaning or investment, or that helped others see something in the design

Creating designs that looked like things from their lives

Although the software has lots of constraints to help students learn the targeted concepts, the students created a wide array of designs. In fact, although DigiQuilt only supports the design of quilt blocks, students made designs that represented a wide range of subject matter (see Figure 33) including fish, dogs, birds, made up animals, people, videogame and cartoon characters, vehicles, houses, trees, and even a hot tub party all in patchwork! Students at both schools in all of the classrooms created quilt block designs that looked like things from the real world. Many students attempted to create houses, robots, people, and letters. One student who was particularly dedicated to creating designs that resembled things in the world was Joanna (who created the house design shown in Figure 24). She created several house related designs and seemed to enjoy helping other students find real world names for their creations or set design goals involving real world items.

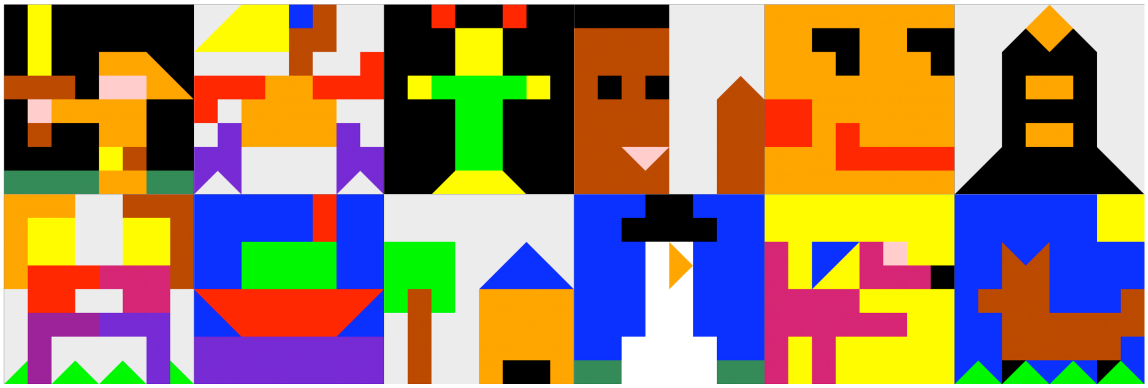


Figure 33. A video game character, a bird, a mouse, two different faces, a rocket, people dancing, a boat, a house, a snowperson, and two different dog quilt blocks – all designed by GW Elementary students.

<u>Creating designs that held special meaning or stories</u>

Lots of the designs had interesting names or stories that went along with them – often without the name or the story it was not obvious what exactly the child had attempted to create. For instance, Lisa and Emma discussed a story at length one day. The story involved a princess and the princess was unable to do what she wanted to do because of some oppressive force. On that day, Lisa created a design called "Hope Broken" (see Figure 34). The design looks like a crown, but the story behind the design was more meaningful for Emma and Lisa as a result of their discussion.

Figure 34. Lisa's design named "Hope Broken"

Sometimes, the students gave their designs names that were a sentence rather than a word or a couple of words. Some of the students seemed as though they would appreciate some other way of connecting their stories with their quilt blocks more directly. Several students made special “friendship quilt blocks” on the last day of their DigiQuilt use. These quilt blocks expressed a certain dedication to maintaining friendships that impressed me.

<u>Naming designs in ways that suggested personal meaning or investment, or that helped others see something in the design</u>

Students expressed themselves through the names they gave their quilts. Naming the designs in a meaningful way seemed to be a regular occurrence. As I mentioned, there were 20 times when naming was the topic of what was happening – students were discussing with each other what to name a certain design or what it looked like. However, that number does not capture the fact that many students named each of their designs carefully or that many of the names held a deeper meaning for the students who created them. Wendy spent the first few minutes each day she used DigiQuilt looking at her printouts and writing their names on them so she would remember the names. Wendy was particularly interested in the naming process for her quilts and often began with a name rather than naming a design based on what she saw.

Wendy’s “Moo Cow” design

> [GW4-5-20-04-t2][Sharing][STACCA][Design][Design idea or plan]
> (18:45) “I’m making one called ‘moo cow.’ We can order 10 more [business cards] today, right?” Wendy wants to order more business cards (though she won’t be in school after this day for the rest of the year, so she won’t be able to get them). She writes an order for ‘moo cow’ before she makes the design.
>
> (21:23) “I’m naming one ‘moo cow.’” She says this out loud, though it is not clear to whom.
>
> (21:50) Wendy asks KK: “What would you call this one?” (The design is now completely black. To view her finished design, see Figure 35.)

KK: “I would call it ‘darkness’.” (Because it was all black at the time.)
Wendy: “I’m naming it ‘moo cow’.”
Douglas: “Why ‘moo cow’?”
Wendy: “Because I love cows.”

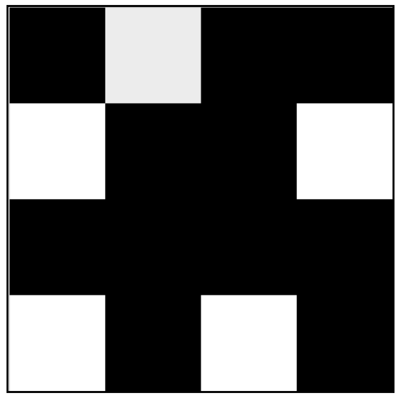


Figure 35. Wendy’s finished “Moo Cow” design.

Some designs (like those in Figure 36) are much more interesting if the viewer knows what they are “supposed to be.” The designs shown here are examples where the names are extremely helpful in helping the audience interpret the intent of the designer.

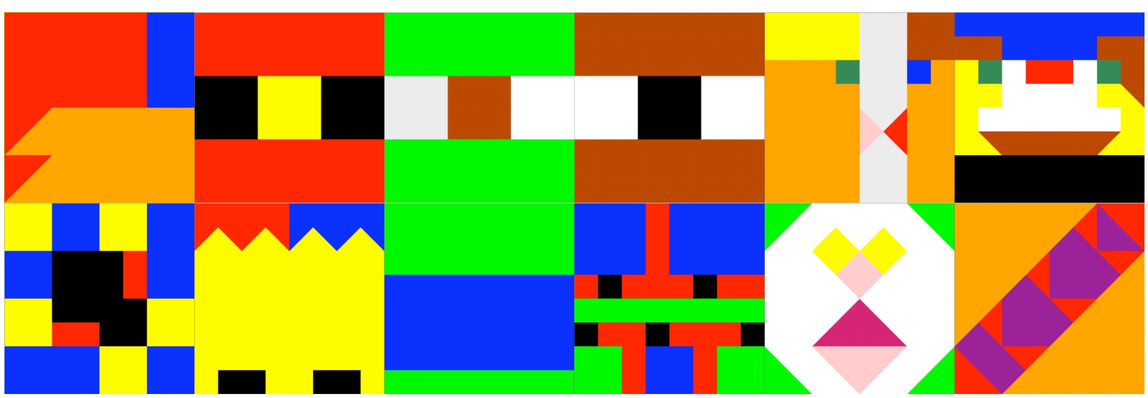


Figure 36. Dr. Pepper, Santa Belly, Dog on Sidewalk, Eye, Lovers, Hot Tub, Skydiver2, Bart’s Close-up, Riverbed, Floor Plan, Ballerina, and Falling Totem Pole – A set of designs whose names help people see something in the design

Sometimes, the students would discuss at length the names they thought their designs or other designs should have. They sometimes started the design process with a name in mind (as in Wendy's "Moo Cow" design), and sometimes looked for something in the finished design to offer inspiration for a name (as in the following example where Peter engages several of his friends in a lengthy conversation about his design).

Peter gets everyone's help to name his design

> [GW4-4-15-04-t1][Naming][Design][How a design connects to something in the world]
> (27:28) Peter talks with Lisa about what his design looks like: a lizard or a dragon.
> Peter: "This one just looks like a dragon." (See Figure 37.)
> His design looks very interesting and has a line of symmetry.
> Peter: (to Lisa) "Wow, look at that, it seems so weird. (He is holding both hands out towards his screen). It reminds me of lizards."
>
> (28:40) Peter asks Austin what she thinks his design looks like. Emma, Austin, and Peter discuss Peter's design (which he had already discussed with Lisa, and which he has just added two patches to since then... one of the patches he added involved three overlapping triangles in order to make it look like the reflection of a similar patch. He spent a full minute just constructing that one patch.). Austin (jokingly) says it looks like the back of a computer to him (since he can't see the screen). Emma (who walks over to see it) says Christmas colors, Lisa says Easter. Austin (tipping the laptop screen so he can see it better from his side of the desks) says, "It looks kinda weird." Peter says he thinks it, "Kinda looks like a dragon." Then, Peter saves his design and names it "Lizard."

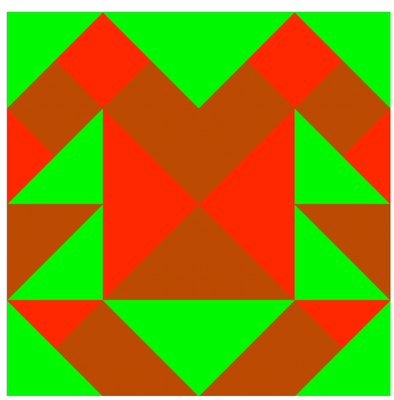


Figure 37. Peter's "Lizard" design.

Though I was hoping students would make a personal connection with their designs and express themselves, supporting those connections turned out to not be the biggest focus of this study. Still, students were definitely making personal connections and expressing themselves in many ways.

The ways children were interested in sharing their designs

It is not surprising that the students were interested in sharing their designs – they made some incredibly creative quilt blocks. The tag combination [Sharing/Telling/Asserting/Commenting or Checking/Announcing][Design] appeared 124 times, with 99 of those being specifically related to sharing a design. In 111 episodes, the combo [STACCA][Design showed up, and 73 episodes there were other kinds of "doing" but not sharing, telling, asserting, commenting, checking, or announcing. So, it seems that not only were students interested in sharing their designs, they did more sharing than any other single activity.

The students enjoyed being on both ends of sharing (for the most part, they were eager to both share and to be asked to share in a design). Sometimes, there was an imbalance of enthusiasm between the sharer and the audience – the sharer sometimes was asked to wait or even ignored or not heard if the intended audience was busy with their own creation.

When the students had the opportunity to have their designs printed on business cards and magnets, they were very excited about it. They spent their concession stand money to buy magnets of their own designs, and they were extremely proud when other students requested their designs or even purchased their designs at the concession stand. Each student was allowed to have business cards of 20 designs initially, with an additional 10 as an option later. The students adopted different strategies for choosing which cards to print – Imani, for instance, printed 20 of his design named "eye" so that everyone could have a copy (see Figure 38), while Lisa chose a wider variety of designs with specific recipients in mind. Wendy was eager to share with me the designs she had collected during break time. The students in GW4 planned elaborate trades in order to get certain designs that they really wanted. The students in GW5 also cared about trading these cards. The biggest controversy was that in that classroom, one of the students' business cards was actually stolen. The students were eager to share their designs, and they certainly saw that there was a "market" for their best designs.

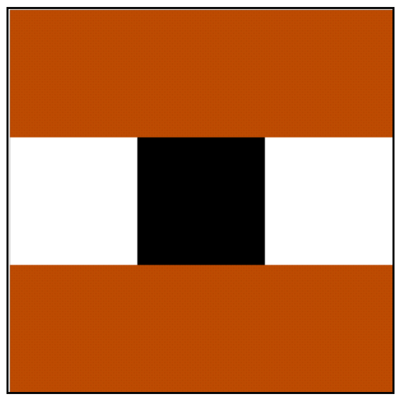


Figure 38. Imani's design named "Eye."

The students were not only sure about which designs they *wanted* to share, but also those designs that they did *not* want to share.

Wendy expresses her dislike of two of her own designs

> [GW4-4-29-04-t3]
> (42:21) Wendy browses her existing designs to decide which ones to request as magnets and business cards.
> Wendy: "I'm not going to do 'red' or 'red two'. I hate those." (See Figure 39 for these two designs.)

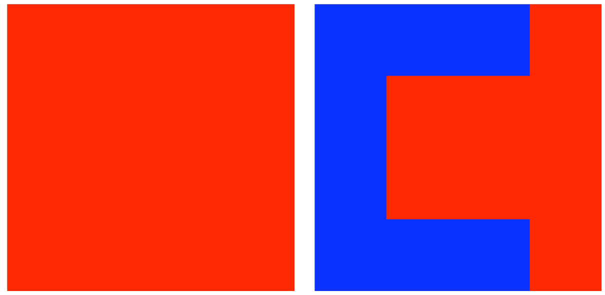


Figure 39. Wendy's designs named "Red" and "Red2."

How children reacted to difficulties

The learners in my study encountered a variety of difficulties and reacted to difficulty in a variety of ways. Some students persisted through difficult moments and ended up seeming very proud of their designs and accomplishments. They compared their progress with each other and explained their progress in terms of strategies or accomplishments that explained how they approached their design activities. Students created goals for themselves that went beyond the challenges offered. Sometimes, students eventually gave up on different aspects of challenges or accomplishing goals they set for themselves that were not required, and sometimes they gave up on an assigned challenge altogether, but that was rare.

The students persisted through more difficulties than I expected they would. Sometimes, they spent up to 10 minutes working on something that never quite worked out how they planned and they started fresh before they finished the challenge, but they still went on to finish (as will be shown later in this chapter when Joanna misses the symmetry in her designs). Sometimes, they struggled with getting the software to cooperate with what they wanted to do (trying to make a shape overlap two patch-holders as Edzier tried to do when he was working on making his design symmetric). However, to me the most surprising times that children persisted were times when they were trying to achieve their own personal goals to make something look a certain way. There seemed to be competing goals: finishing the designs quickly and making each design look a certain way. Oftentimes, making the design look a certain way won out over finishing quickly. In the following example, Wendy continues to follow her design goals to create her "dizzy" designs in spite of her efforts taking a lot of time and not always going as planned. She discusses her goals with Joanna along the way.

Wendy's "dizzy" designs

> (9:55) Wendy: "Oh, I know exactly what I'm going to do. It's going to be so cool. And I'm not trying to copy off Ramona." Wendy continues to explain how she's going to create her new design with nested squares of different colors. Joanna compares it to, "a target, but square." (See Figure 40 for Wendy's completed "dizzy" design.)
> Wendy: "You can do it too, if you want."
> Joanna: "No, I'd rather not copy." She continues working on her design.
>
> (10:20) Wendy: "That would work for one line of symmetry, wouldn't it?" referring to the "square target" design idea.
> Joanna: "Yeah."
> Wendy: "I'm going to call it 'dizzy'."
>
> (10:32) Wendy: "but wait, wouldn't it be kind of cool if I..." she continues to talk about a design similar to 'dizzy' that has alternating colors in each column. She states that she will do it later and call it 'dizzy two'.
>
> (10:39) Wendy: "Joanna, can you come on and do it with me?" referring to the 'dizzy' design.
> Joanna: "not right now."
>
> (13:05) Wendy: "I'm getting dizzy already," referring to her 'dizzy' design. Wendy is using the 16-patch BWA grid as a guide and laying one small square down at a time, tracing the outside square.
>
> (13:14) Joanna watches Wendy place the small squares one by one and states, "Actually what you can do is, one of those [referring to the rectangle piece] equals two of those [referring to the small square piece]".
> Wendy: "Why didn't somebody tell me that before?"
>
> (13:38) Wendy says that she has three ideas for versions of 'dizzy', and that she will name the third one 'dizzy three'.

(14:07) Wendy calls over KK and tells her that she is going to make three 'dizzy' variations. Joanna explains the variation with the alternating colors. Wendy explains the next two variations.

(14:39) Wendy and Joanna talk about how the strategy of using rectangles saves time over using squares.
Wendy: “oh gosh Joanna, that's going to take a long time.”
Joanna states that, “You’ll have to use the rectangle. It’ll save you like 30 seconds.”
They continue discussing how much faster it will be.

(15:40) Wendy continues working on her 'dizzy' design, and as she adds more rectangle pieces, they begin overwriting/deleting the existing laid down pieces. She asks for help.

(16:50) Wendy calls KK over for help. KK explains that it's a software issue and explains how Wendy can work around this problem. Wendy does not seem to understand how the orientation of the patches affects the placement within the patch-holders.

(18:05) Joanna asks KK how to position a rectangle in certain place. She is having the same issue as Wendy. KK explains how to rotate the patch to orient the rectangle in the correct position.

(20:20) Wendy jokes that she should just fill in the middle with pink. When Joanna
Elizabeth asks if she is going to do that, Wendy says, “No, that would ruin my whole design.”

(21:36) Wendy is still having problems with her first 'dizzy' design.

(22:22) Wendy: “Oh my gosh Joanna, this is going to be so hard to do all three.” regarding her dizzy design sequence.

(24:15) Wendy begins placing the third inner square of her dizzy design.

(25:01) Wendy tries to place a single large green square as the center (bull's eye) of the square target. The square will not snap into the center because of the way the block is partitioned. The large square snaps over and deletes some of the smaller blue squares the Wendy has placed. Joanna suggests she change the grid type. Wendy does so and it does not help. Wendy realizes the problem is that the base block does not have a position for a square right there. Joanna then suggests that Wendy add four small green squares for the center rather than one big one. This idea works and Wendy works on finishing her design.

(27:30) Wendy finishes 'dizzy' and saves it.

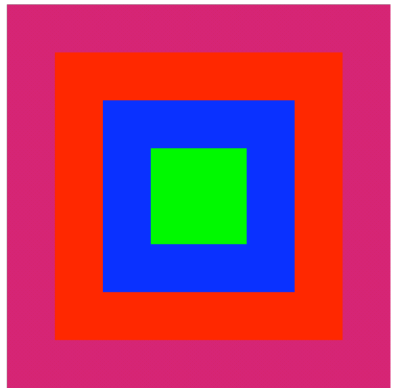


Figure 40. Wendy's first "Dizzy" design.

Wendy continues with her series of dizzy designs, moving on to the second variation she had planned. After a while, she decides that her efforts are not resulting in fast enough progress.

(39:20) Wendy: "You know what Joanna, I'm just going to start filling [the second "dizzy" design] in. It's too hard." Wendy begins filling in the block with random shapes. However, even this proves to be difficult since she continues to try to use rectangles that line up in unexpected ways.

(40:15) Wendy: "Oh my gosh, this is hard." Wendy asks KK for help. KK helps her quickly, and then Wendy continues working. Wendy finishes her second "dizzy" design and saves it (see Figure 41).

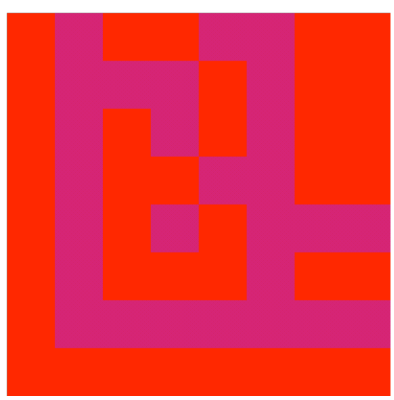


Figure 41. Wendy's design named "Dizzy 2"

Wendy spent 30 minutes making these two "dizzy" designs. This theme was exciting to her even after all of that. In fact, she enjoyed this challenge so much that she revisited her series of "dizzy" designs on the last day, constructing one more design for the series (see Figure 42).

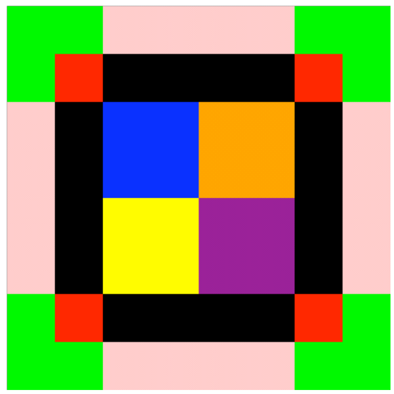


Figure 42. Wendy's design named "Dizzy 3"

Like Wendy, the other children made design challenges for themselves on top of the challenges that were given to them. There were often simpler ways to solve the challenge that would not have taken much effort (for some of the students, they would have been so easy that they could have finished all of the challenges in minutes). Rather than pursue the simplest approach to solving the challenges, the students often attempted to solve the challenges with additional constraints they added for themselves. Many times, their additional challenges made it difficult to succeed, but the students persisted in spite of these difficulties.

One kind of challenge that students in GW4 did not encounter or find on their own was the idea that they could make their designs appear animated. This activity was discovered by several 5th grade students and became popular at CH elementary. The students discovered that by using the swap, undo, and redo buttons, they could make their designs appear to move. Several students adopted this goal even though it could be difficult to achieve the desired effects. The students explored the use of the swapping tool in conjunction with the undo and redo buttons to achieve this goal. Part of the reason this challenge was so popular seemed to be that it was hard and the results could look very interesting.

Exploring children's math talk and "bridging"

The second set of predictions related to the hypothesis, the "bridging" predictions, state that children will connect their concrete quilt blocks to the abstract mathematical concepts (in this case fractions and symmetry). The predictions related to bridging include:

1. Children will connect the abstract ideas of "school math" to attributes of their (concrete) designs.

Indeed, the data shows instances where children seemed to make connections between a symbolic fraction and a concrete representation of a fraction or between the idea of reflective symmetry and an instantiation of this type of symmetry. My predictions did not stop at saying that the children would make those connections. I further stated that *we will help children make these connections* in two ways:

2. By helping them view their artifacts in a mathematical way, and
3. By allowing them to interact with the artifacts using tools that help them reflect on their artifacts in a mathematical way.

The children talked about fractions and symmetry throughout the study. Their conversations showed that students had both a drive to understand and a drive to explain. At times, they were able to leverage the software tools to help them meet both of those needs. Other times, it appeared that the social system played a stronger role in helping the students connect the abstract and concrete. The interplay between the social and technical aspects of the system was often complicated. To inform this hypothesis more completely, I need to show more than just that connections were made – I will present the data and describe as much as possible about what supported the children as they made their attempts at bridging. Then, I will compare and contrast the bridging examples for fractions and symmetry – I believe that there is a story there that I had not anticipated, and that this story can inform the design of similar computational manipulatives.

There were interesting differences between fractions and symmetry bridging. This was not just a matter of using the same process with different content – the different ways the same tools were used for fractions vs. symmetry combined with the different types of peer and teacher support that were sought suggest that the learners approached these content areas in very different ways. It seems that they wondered about symmetry in a more interactive way than fractions, using gesture and conversation more readily. Their exploration styles for symmetry were more active and externally observable than for fractions.

To show the impact the tools, activities, and modes of interaction that were provided had on children's ability to make those connections, I will show examples where there seemed to be a correlation between a child attempting to connect a design to the concepts of fractions or symmetry and the use of software tools, the guidance provided by the challenges, or the act of constructing a design in this thoughtfully constrained environment. I will also show examples of times when I thought that a child missed an opportunity to make a connection and describe possible reasons or possible supports or software design choices that might be used in the future to avoid such circumstances.

Examples from the video data from GW4 will show, in context, times when children made these connections. Then, I will pull examples from the other classrooms' video data in order to show that these types of connections happened in other settings too. The examples will show times when children said or did something that indicated an attempt to bridge between the abstract ideas of symmetry or fractions and the concrete quilt block designs they constructed.

Fractions Bridging

The students talked about fractions throughout their DigiQuilt experiences. Of the 312 episodes (361 episodes and sub-episodes) from the GW 4th grade dataset, there were 23 separate episodes where fractions were involved in a visible way (31 episodes and sub-episodes). In 21 of those episodes, a learner seemed to be connecting the symbolic fraction to the concrete examples, while in 2 episodes, there seemed to be a failure to make a connection. That means that over the 6 days on which video data was successfully collected (recall that on the first day, there was no sound on the video), the video captured an average of 3.5 interactions per day where fractions were a topic of the mathematizing that occurred.

There was one example episode from the GW4 dataset that I think embodied most of the kinds of supports that successfully supported learners as they attempted to bridge between their designs and fractions. This episode was presented in the chapter 5 day-by-day accounts of children using the DigiQuilt system, but I will present it again for simplicity's sake. I will follow up with similar episodes from GW4 as well as from the other classrooms in order to show that although those examples are particularly rich, the types of things that happened in great quantity in this episode happened elsewhere as well (if sometimes less frequently).

Emma's fraction exploration

(20:51) Emma (to Lisa): "What's another name for 15/32s?"

Lisa and Emma have a conversation about this. They write some guesses and calculations on paper.
Emma: “How many [DigiQuilt] squares is that?” (Emma counts and mutters.) “Because that’s the question that I’m on.”

(21:49-23:13) Emma is placing big squares in a diagonal pattern in the block work area. After she places the 4th square, she says, “1/4,” which she seems to be reading from the color button on her screen. She continues trying to get 15/32 by adding shapes to her screen, but she is not successful.

(23:13) Emma: “Lisa, I’m trying to understand this thing, but I don’t get it.” Emma asks Lisa again about the fraction. Emma adds, “I got nothing,” expressing her lack of understanding. Lisa and Emma try to figure out another “name” for 15/32.
Lisa: “Maybe it’s 1/16.” Emma adds some squares to her block work-area and says she doesn’t think so because then that would be “it.” Emma then suggests maybe it *is* 1/16 and asks, “What's another name for 1/16?” Lisa writes on paper and tries to figure it out and comes up with 2/32 and 4/64. Emma continues to struggle with the fraction. Lisa goes back to work on her task (she is not working on the same challenge as Emma).

(25:24) Emma says, “Lisa, it’s not on here.” She keeps adding pieces to her block work area and then reading the fractions from the fractions-feedback aloud.

(25:52) Emma: “It can't be 2/32”
Emma asks the teacher: “What's another name for 15/32”?
Ms. S asks the girls if they know a way to find an equivalent fraction.
They both answer, “divide it”. This conversation continues. Eventually, I clarify that you can't *reduce* it anymore (the conversation includes some mention of the idea that you can still come up with another name for it by multiplying).

(26:48) Emma and I discuss how 15/32 is close to 1/2, but not quite the same.

(27:36) Emma has definitely figured out a strategy for finding 15/32. She is adding rectangles to her screen one at a time. Since each one is 1/32, once she has 15 of them, she finds the answer!

(27:56) Emma: “I found it!” (She hides her computer screen.) “I'll tell you what it is.” Emma covers her screen and tells Lisa she can't see it because Emma doesn’t want her to. Emma added that she, “[doesn't] want Lisa to know”.

Recall that this episode took place over the course of about 7 minutes (longer than usual for Emma, who was a very prolific designer). Emma struggled to solve this problem, and she used every tool available to her. In spite of her difficulties, she did not give up. The feedback from the software helped her solve the problem eventually, but it was not the only source of support for Emma to make this connection between a concrete example of 15/32 and the symbolic fraction in the challenge. Emma used a variety of strategies to find a solution for the problem – some methods she recalled from her previous experiences with fractions (e.g., multiplying to find equivalent fractions), some support from friends and teacher figures, and some support from the software.

Different supports for fractions bridging – examples and analysis

In the following set of examples, some subset of these supports helped each child bridge between a fraction and an instance of that fraction expressed in a quilt block. It is interesting to note what the children wonder about in each case, and how they address their wonderings.

Learners' discussions about fractions sometimes took place in the abstract, meaning they were trying to use algorithmic approaches to figure out some fractions related concepts (like finding equivalent fractions) rather than always using the software to support their problem solving. Their success with remembering how to operate on fractions to figure things out varied. Sometimes, as in the example "Emma's fraction exploration," a series of non-technical supports (consulting with a peer, the teacher, and me; writing calculations; or reflecting on what the challenge is asking) is interleaved with technical supports (like the fractions-feedback). In cases like that, the social support seemed to keep the student going – Emma kept trying to find out more about 15/32 even though it was not a simple task for her. In the end, it seemed like the fractions feedback in the software helped Emma solve the challenge, but the support for her fractions explorations came from many sources.

In fact, Emma was not alone in her struggle to figure out 15/32. The next example follows an episode where Austin is using the fractions-feedback to figure out 15/32 (which will be described in the next subsection – a connection between the abstract and symbolic that seems to be supported by a tool in the software). Once he figures out how to solve the problem in a very basic, boring design, he begins making a complex, symmetric design that has 15/32. He wanders around the classroom for a bit, I think looking for someone who wants to see his design. He is beaming and seems quite proud of his accomplishment. He shares his progress with Peter. In the end, they discuss ways to cover the design so it has 15/32 of a color.

Austin's fancy 15/32 design

> [GW4-4-29-04-t1][(33:40)]
> Austin has figured out the 15/32 challenge. His design is very intricate and has a line of symmetry. He was walking around (I think trying to find someone to show it to). He turns to Peter and says: "Do you know what 15/32 is? I figured out number 4 [the 4th challenge]. Ok. One of these [points to something on Peter's screen] is 1/16. [Pause for a couple of seconds] Want me to tell you what 1/15, I mean, 1/32 is?"
> (Peter is difficult to understand on the tape, but I think he says 7 of something (and he may say one of the rectangles in addition to that)). Austin replies, "No, this is 1/32, so 15 of those."

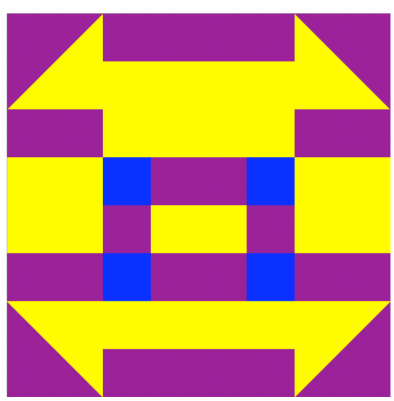


Figure 43. Austin's design that shows 15/32, 15/32, and 1/16.

In this example, Austin seemed eager to share his new knowledge about how to get 15/32. Peter did not seem particularly impressed, and in fact suggested another way to get 15/32 that Austin did not seem to attend to. However, it is interesting that Austin cared enough about solving the problem to share his newfound knowledge about fractions (and his design) with a peer. Austin attempted to share with Peter a connection between the fraction 15/32 and a design strategy in DigiQuilt (use certain shapes in order to reach a fractions goal).

Sometimes, the learners' math talk was fairly one-sided – the learner did not really try to engage others in a dialog about the math at hand. Rather, the learner might simply state a new fact they have discovered, assert that something they have done solves a challenge, comment on a design or a mathematical aspect of a design, or just seek confirmation that something is correct. In the following example, Imani asserts a fractions fact.

Imani's fraction assertion

> [GW4-4-22-04-t1][(24:00)]
> Imani looks at a design that is half blue and half white. He says that 1/2 is 8/16. I ask him how he knows that and he explains that half of 16 is 8. Then, he shows me the line of symmetry in his design.

Imani seemed proud that he knew that these fractions were the same. He was seeking verification that his design solved the challenge, and he was ready to show me how he thought it did just that. In this case, the student is correct in his assertion that ½ and 8/16 are the same. Imani's design is showing this fraction, though it is tough to say what inspired him to make this connection between ½ and 8/16. In fact, the challenge involved the fraction 8/16, and the fractions-feedback would have been telling him ½, but he did not indicate if that influenced his thinking. It seems likely, however, based on the dialog and the challenge at hand, that the challenge was involved in supporting the connection he made (the possible role of the software is less clear).

There was some discussion of "other names" for fractions at least 5 times. I asked Edzier for another name for 3/8, Emma asked Lisa for another name for 15/32, Imani asked me for another name for 5/16, I pointed out the fractions feedback to a student who didn't see to see it yet but who was working with a challenge where it would help, and Imani also asserted that 8/16 and ½ were the same.

These examples showed children making or sharing connections between the symbolic fractions and their designs where there were a variety of social supports playing a role in the bridging that occurred. There were seven episodes that included discussions and guiding questions or feedback from a teacher figure. Sometimes, students discussed fractions with their peers or asked fractions related questions aloud to whoever was listening. There were also some episodes where children were talking about fractions or just mentioning fractions and it was not obvious the context or inspiration that led them to talk about fractions. In all of these cases, students were mathematizing their quilt designs at some level, and that was exciting to see.

<u>Software support for fractions bridging</u>

Of the 23 episodes included in the pool of episodes where children are connecting (or attempting to connect) the abstract idea of fractions to their concrete quilt blocks, 13 seemed to involve the fractions-feedback in the software and 3 used the grids for help with fractions. Although the software was not directly involved in all of the fractions talk, the software tools seemed to play some role in the conversation or in helping a child connect the abstract to the concrete in several other instances.

Fractions-feedback

At least two interesting episodes involving the fractions-feedback occurred in the context of a challenge involving the fraction 15/32. In the software, when a child is using a 16-patch block work area, the half-square triangle and the half-square rectangle are the two shapes that, by themselves, can be used to cover 1/32 of the area of a quilt block. Understanding how to make 1/32, then, can be one pretty important part of solving the challenge (there are ways to solve the challenge without figuring that out, but when starting from a blank block work area, as the students often do, 1/32 is a good place to begin).

The previously mentioned examples, "Emma's fraction exploration," and, "Austin's fancy 15/32 design," both involved the tricky fraction 15/32. I've already described Emma's struggle, but the example with Austin was just about his sharing of the connection he made – now I'll share the example where he actually solved the challenge. In the following example, Austin tries to make a quilt block that is 15/32 covered by a color. He adds shapes to his design and watches the fractions-feedback on the color buttons. I intervene to ask him some questions about what he is doing. My questions highlight the capabilities of the software to help him figure out a way to find 15/32.

Austin finds 15/32

> [GW4-4-29-04-t2][(9:45-10:37)]
> Austin begins again by adding triangle pieces that are half the size of a patch.
> KK: "So, how much does each triangle cover out of the whole quilt block? What fractional area does it cover?
> Austin: "Uh, I mean, 1. Ummmm, half of a 16th, so 1/8th… um, no, darn."
> KK: "When you add one, how much does it change the fraction over here [gestures to the fractions-feedback]."
> Austin: "Um, by ½. By one? Hold on."
> KK: "When you added the first one, how much was it?"
> Austin: "Um, one 32? So, if I add 15 of those, the-en yay!"

When asked to predict how his actions will impact the fractional area of the quilt block that is covered in a particular color, he first tried to use an algorithm in his head (dividing by 2) to figure out what the fractions would be. Connecting his idea to divide by 2 to the fraction was difficult. He said he wanted half of 1/16, so 1/8. He divided 16 by 2. Through this process of predicting and then testing what actually happens, he stepped through the process of constructing the answer. The feedback offered a visual aid that related to the actions he took. By saying his predictions out loud, he let us into his mind a bit more than if we did not have this shared, external representation of his actions. Of course, my intervention probably supported Austin's learning as well. It is nearly impossible to find an example where the social part of the system did not play a role.

The fractions-feedback was used in 9 episodes where learners restated the fractions from the fractions-feedback and related the information to the challenge. In 7 episodes had learners connecting the shapes they were using to construct their designs with the fractions-feedback. That made a grand total of 13 episodes involving the fractions-feedback in an obvious way (3 episodes fell into both categories mentioned above). Evidence suggests that the fractions-feedback was also utilized in many cases where the interaction was less obvious or could not be observed directly.

Lisa discovers fractions-feedback

> [GW4-4-22-04-t3](9:15-9:25)
> Lisa: "oh, cool. Right here [points at the color buttons] it shows you how much of your screen is each…"
> Emma [interrupts]: "yeah, that's what I've been doing."

The only evidence we have that Emma knew about the fractions feedback before Lisa is from one minute before Lisa shares with Emma about the fractions-feedback, so we can't be sure she knew before that. This example emphasizes that while we can't always be certain when a student is using the fractions-feedback, we do have evidence that several students found it and used it. Students certainly noticed the fractions-feedback when they had challenges that involved fractions that were not reduced – they found that they could not get the fractions-feedback to say numbers like 8/16, even if that symbolic fraction was in the challenge (as in the following example). Imani did not seem confused by the mismatch between 8/16 from the challenge and ½ that he was seeing in the fractions feedback, but there were examples from both schools relating to the challenge to find ½, ¼, and 2/8 that revealed that some students' attention was captured by this mismatch between the fraction 2/8 and the feedback stating ¼.

When the use of a tool involves some interaction or some visual change to the screen, it is possible to tell that a child is using that tool. In the case of the fractions-feedback, it is sometimes difficult to spot its use because neither of these clues to discern tool use is present. It was only obvious that the feedback was being used if the learner repeated something they read form the feedback (9 times), or if they were clearly connecting the feedback to the shapes they were adding and taking away (7 times).

The select-a-grid tool

The select-a-grid tool, which was developed with fractions learning in mind, was used three times (in the 312 episodes) for supporting students in bridging between symbolic fractions and concrete instantiations of those fractions. Two of those times, it was used for construction and the designs remained unchanged once the learner understood the equivalence of 8/16 and ½. The third example shows a different approach.

In this example, the select-a-grid seemed to help Lisa work out a similar challenge involving equivalent fractions. The challenge was, "Make a quilt block that shows 1/2, 1/4, and 2/8." Using the windowpane-like grid that divides the block work area into 4 pieces, she first put 4 large squares in to fill 1/4, and then she seemed to have an ah-ha moment as she sat up straighter and sharply inhaled, cleared her design, made a new one to solve that challenge very quickly, and moved on to another design. She said (seemingly to herself), “Oh! I get it. Gosh that’s easy. I’m so stupid.” Because of the rapid flow from one activity to the other, it seemed like she used the grids and the feedback together to help her understand the challenge. What is unique about this example is that the resulting design is much more intricate than the initial design she made when she was experimenting with big pieces and using the grid in conjunction with the fraction feedback (see Figure 44). Many students who used the select-a-grid tool did not take extra steps to make their designs more intricate.

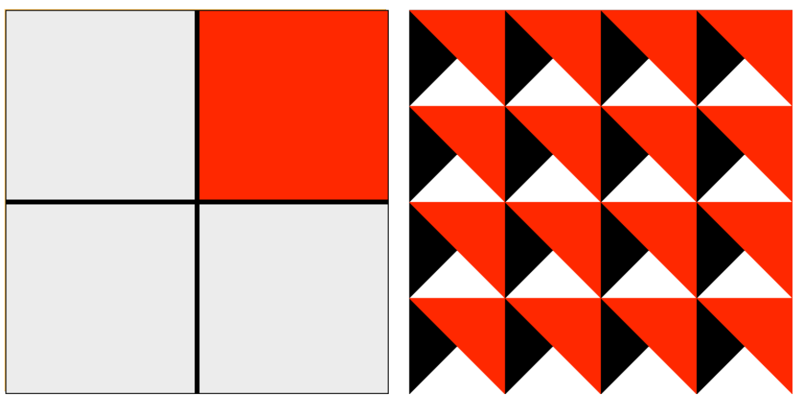


Figure 44. The design Lisa made using the select-a-grid tool and the design she made once she realized that 2/8 was the same as ¼ (the design is named “Evil Mountains”).

<u>Experiencing difficulty connecting an abstract fraction to a concrete example of that fraction – connection not made</u>

Finally, I will describe episodes where children seemed to be experiencing fractions related difficulty that went beyond asking for and receiving help or figuring things out. There were 2 such episodes. Students did not seem to experience unresolved difficulty with fractions very often (difficulty experienced by the students that they either did not notice, did not work through completely, or gave up on trying to understand). I will share both examples because each is unique in some interesting way.

The first example shows Lisa come to a point where she thinks she has solved the challenge, but she has not actually done so. Lisa spent a lot of time on the first three challenges (which Emma pointed out to her several times – comparing their progress throughout the time they spent using DigiQuilt). Toward the end of their time using DigiQuilt for the day, Lisa begins the 4th challenge (the same challenge that Austin and Emma struggled with – 15/32). She places small pieces (rectangles and small squares) into a blank block work area until the feedback says 25/64. She says, aloud but seemingly to herself, “25/64.” She pauses for a few seconds and does some quick calculations on paper. Then, she whispers, “Yes! I did it.”

Unfortunately, it seems that Lisa convinced herself that 25/64 was the same as 15/32. Most likely, she was just rushed at the end and wanted to finish one more challenge. She participated in much of the discussion with Emma (as outlined in the previous subsection), but she was working on an unrelated design at the time and so lacked the benefit of working it out at the same time as Emma and seeing the impact of adding a rectangle to a blank block work area. Lisa probably did not even realize that she was having difficulty.

As mentioned in chapter 5, Edzier, on the other hand, was quite aware of his difficulty. As he attempted to solve the challenge to, “Make a quilt block that has one line of symmetry and is 5/16 some color (fill in the rest of the design with other colors),” he worked for over 15 minutes. He seemed to understand what he was trying to do, but he was not able to accomplish his goals in the end.

Edzier’s double challenge (as included in chapter 5, but more details here)

> (0:00) Edzier works on the first challenge. His design looks like the letter “h”. He raises his hand after he finishes. He shows me his design. He says he doesn’t get

number 1. I tell him that it looks like he has 8/16 red and 8/16 yellow, but he needs to have 5/16 of some color.

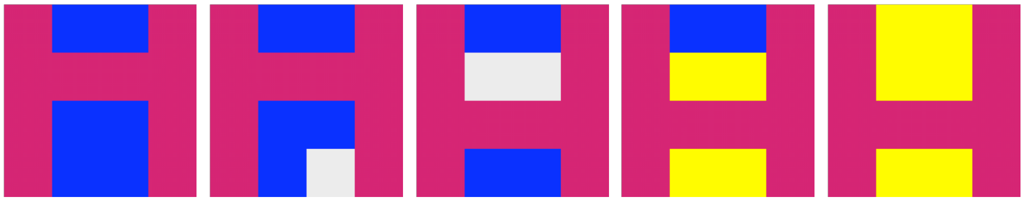


Figure 45. In the first two minutes, Edzier changes his design several times, raising his hand between each try, but not receiving any feedback until after these 5 tries.

Edzier changes his design so that it is 5/16 red, 5/16 yellow, and 3/8 green. His original design was symmetric, but now it is not. What began as a difficulty with fractions turns into a difficulty with symmetry. Edzier continues to try to solve the problem.

Figure 46. Edzier now has the fraction 5/16 in his quilt, but his design is no longer symmetric.

(6:00-7:40) Edzier clears his design without saving. He puts in 4 green squares and raises his hand. He adds more shapes. He raises his hand again and then clears his work again without saving. He rests his chin on the laptop in the space between the trackpad and the keyboard and keeps working.

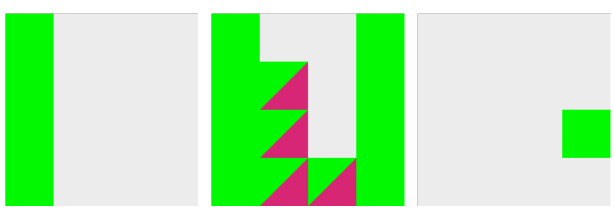


Figure 47. Edzier tries several more designs.

(8:40) Edzier raises his hand again. He glances at the camera. He looks either very tired or very frustrated. He still doesn't get it. I ask him if he wants to skip it and come back to it, and he shakes his head. I ask, "How much is covered now, with the green color." Edzier correctly replies that 1/16 is covered. "Do you think there is a way you could cover 5/16 with green?" Edzier puts in 4 big squares. I explain that he has to be careful when he puts in the last 16th because he needs to keep a line of symmetry. He tries to set a patch-sized green square overlapping the edges of two snaps in the block work area, which doesn't work. I tell him he'll need to use two smaller pieces if he wants to put it there. At this point, I think Edzier will get it soon, so I leave him to help other students. He switched between the designs in Figure 48 several times, always trying to place the 5th green patch so that it overlaps two patch-holders.

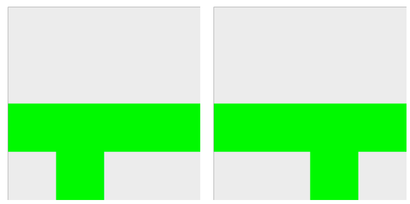


Figure 48. The two designs that Edzier switches between several times.

(10:47) Edzier has selected a grid that splits his block work area into 8 pieces and has a vertical line (seems to be the line of symmetry he is after). He repeatedly picks up the same green square and tries to set it on the line, but it always snaps one way or the other (See Figure 49, left and center.). Then, he uses smaller rectangles (but since he already has 5/16 of the quilt block covered in green, this does not help him solve the challenge). He raises his hand to ask for help again. The piece that won't land where he wants it needs to be removed before this new idea he has come up with will work. He is very close, but cannot seem to figure out how to make the design symmetric and still have the fraction 5/16 (See Figure 49, right).

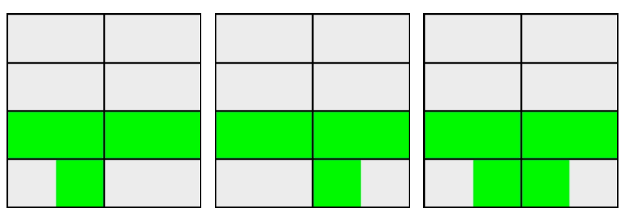


Figure 49. Edzier's designs as he approaches a solution.

(10:52) I ask him, "How many 16ths are covered in green." He replies, "3/8." I ask, "How many 16ths it that?" He answers correctly. He removes both of the green squares from the bottom row, and as he does that, I point out that when there is just one of those squares left, it was 5/16 in the fractions-feedback part of the screen. I suggest that he use 4 small squares to fill in his design in a symmetric way. He adds the squares in a way that maintains the symmetry (See Figure 50).

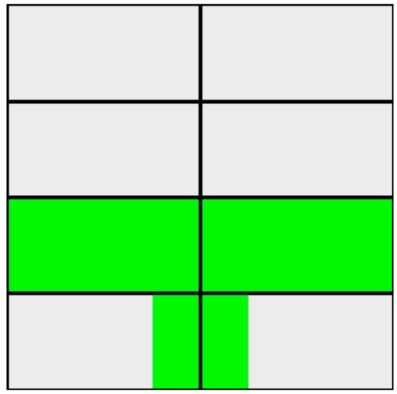


Figure 50. Edzier's design right before he finishes it by adding other design elements.

Unfortunately, his design still has blank spaces that contain no pieces. I suggest to him that he fill in the rest of his designs with other colors.
(11:40) He fills in the rest of his design and saves it as "design1", but the way he fills it in is not symmetric, and it includes green, which makes it so that the quilt block is not 5/16 green anymore (see Figure 51 to view Edzier's design with the grid as he had it and without the grid (as it would have looked in the block browser)).

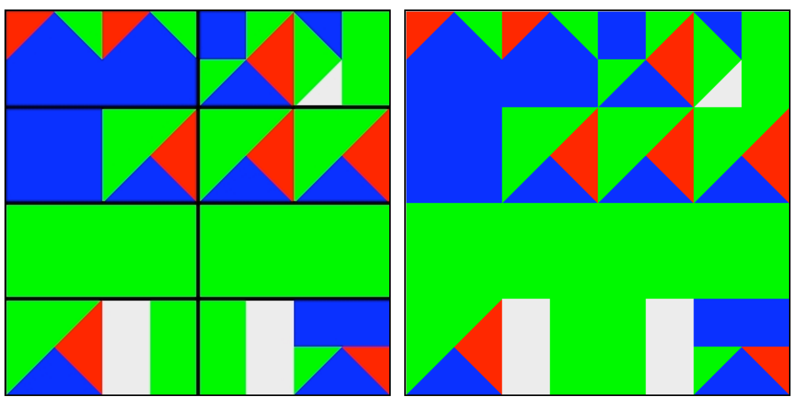


Figure 51. Edzier's design that is neither symmetric nor an example of 5/16

(15:23) Edzier raises his hand for a moment. Then, he saves his design and clears it. It was not symmetrical, and it did not have the right fraction anymore. In my opinion, he looks very tired on the tape, and I think he honestly just wanted to move on to something else. The fractions-feedback would not have been telling him he had 5/16 green, so he probably made a conscious decision to move on in spite of the design not quite solving the challenge.

As mentioned in chapter 5, one possibility for preventing other children from experiencing this difficulty would be to get children to save several versions of the quilts as they work through them. This might be one way to help them become less attached to any one solution. This design solution assumes that if Edzier had several examples that contained 5/16 green and several that were symmetric, he would be able to figure out how to make one design that would have both attributes. It really did seem as though Edzier understood these two problems separately, but that somehow combining the two challenges into one quilt block was too much for him. The way he was attempting to make his design symmetric suggests that he realized how much green he needed to add, but that he was unable to understand how he could add that amount of green in that location (a location that would maintain symmetry). Edzier relied on his knowledge of symmetry when that was his focus, but once he finally added the correct amount of green in a way that was symmetric, he stopped paying attention to the shapes he was adding, and even added more green, so his design went from solving both parts of the challenge to solving neither part.

Sometimes, as with Edzier, the students had a tough time solving the challenges that were provided. Even when the challenges were not ones they came up with on their own, students often persisted for a surprisingly long time. In this example, Edzier struggled to create a design that showed the fraction 5/16 and had one line of symmetry. He was willing to struggle for a long time before he eventually did give up. This example was one of the episodes that made me realize that oftentimes, the examples where students experienced difficulty were closer in character to the rich success stories than they were to the shorter supporting episodes. The element that connects the two different kinds of episodes is the element of the struggle. The only thing that sets apart the rich examples from the examples where the children experienced difficulty is the one moment where things eventually clicked. In the examples where students struggled and eventually moved on without succeeding, we can see many elements of the socio-technical system at work, and we can get a sense for which supports were missing. Compared to the rich example episodes, this kind of episode tells us as least as much about the kinds of supports that students rely on when they are trying to connect the abstract and concrete.

For Austin, we saw him first struggle to understand the fraction 15/32, then we saw him share his knowledge of 15/32 with his friend Peter, and finally we saw him create a really intricate, complicated design. For Emma, we saw her struggle to figure out 15/32, and when she moved on to create a design using her new knowledge she seemed to forget what she was doing. She never created a design that she liked that included that fraction. For Lisa, we saw her struggle with 15/32 along with Emma even though she was not on that challenge. Later, we saw her convince herself that 25/64 was the same fraction.

Each of these examples points out a different part of the socio-technical system that worked or didn't quite work for the student in question. For Austin, the fractions feedback and guidance from a teacher helped him figure out how to make 15/32, then he wanted to share his new knowledge with his friend Peter, and finally, he was motivated to create a design that was intricate and still solved the challenge (a design which he later printed as a magnet and on business cards). For Emma, the social elements of the socio-technical system seemed to support her through her struggles to figure out 15/32, which she eventually came to understand using the fractions feedback. For Emma, the socio-technical system might have supported her better as she endeavored to use her knowledge to create a design she cared about more deeply. Lisa ended up solving the 15/32 problem incorrectly. She was rushed at the end of the day to finish. Perhaps she would have been more successful if the system allowed her to pause in her work more easily so she could have explored 15/32 at the same time as her friend Emma.

For both Emma and Austin, Their struggles to understand 15/32 were long, but resulted in success. For Edzier's double challenge, his struggle was long, and though it ended without complete success, he worked to solve the challenge for quite a while before giving up. The only thing missing for Edzier was that final "ah-ha" moment where everything fell into place. All of these students persisted in spite of their difficulties along the way. Even Lisa persisted until she *thought* she was successful.

Symmetry Bridging

Generally speaking, the children in the GW4 class talked a fair amount about the symmetry of their designs. There were 48 episodes involving symmetry (out of 312 total episodes). Of those 48 episodes, there were 5 that did not seem to have any bridging or connecting related to symmetry. That means there were 43 episodes where students seemed to connect the idea of symmetry to concrete instantiations of symmetry – that averages to approximately 6 per day that were captured on video.

There was one example episode from the GW4 dataset that I think embodied most of the kinds of supports that successfully supported learners as they attempted to understand symmetry. This episode was presented in the chapter 5 day-by-day accounts of children using the DigiQuilt system, but I will present it again in order to highlight even more of the detail about how the bridging seemed to be supported and how much support was needed.

Peter's Symmetry confusion

Peter took about 11 minutes to work through his confusion entirely and create a design he found suitable for solving the challenge. It was his first design of the day on day2. I have included the tags here as well as a sample of how tags would be applied to a rich episode such as this one. Note that this episode was only counted as one episode where any of these tags were applied if I was counting episodes, but any time where I was counting the number of times a tag was used, it would count each time a tag was applied. Note that both new and old tags are still included.

[GW4-4-15-04-t1][Talking math][Symmetry]
[AH][Symmetry]
[AH][Challenge(s)]
Asking/helping about targeted math concepts – symmetry
Asking/helping about challenges
(6:12-12:40) Peter is very confused about what the challenge is asking. After his neighbor reads the challenge aloud, he keeps repeating, "one line of symmetry," and questions relating to how that is possible. He keeps asking his neighbors and talking to himself trying to figure out how the quilt block could possibly only have *one* line of symmetry. Lisa explains several different times. Lisa sometimes replies right away, and other times she keeps working or asks Peter to wait. Eventually, Peter, "gets it."

> [GW4-4-15-04-t1][WLB][Symmetry]
> -**Watching someone else work on something that the watcher is struggling with**
> (6:50) Peter watches Lisa.
>
> [GW4-4-15-04-t1][AH][Challenge(s)]
> [AH][Symmetry]
> -**Asking/helping about challenges**
> (7:24) Peter asks Lisa for clarification about the challenge.
> Peter: "are you doing one?"
> Lisa: "yeah"
> Peter: "one line of symmetry?"
>
> [GW4-4-15-04-t1][AH][Symmetry]
> -**Asking/helping about targeted math concepts - symmetry**
> (8:10) Peter looks at Lisa's design so far and asks her "won't there always be one line of symmetry?" Then, he motions to the horizontal and vertical axes of Lisa's quilt block, saying, "There will always be that one and that line of symmetry." Peter goes back to his design.
> [GW4-4-15-04-t1][Checking][STACCA][Others][Progress]
> -**Wanting to see what someone else has done/checking someone else's work**
> (8:58) Lisa looks at Peter's screen.
>
> [GW4-4-15-04-t1][WLB][Symmetry]
> -**Watching someone else work on something that the watcher is struggling with**
> (9:00-9:10) Peter is continually talking to himself and intermittently looks at Lisa's screen.
>
> [GW4-4-15-04-t1][DD][Symmetry][of a design]
> [AH][Symmetry]

[DQT][grid][Symmetry][of a design]
-**Asking/helping about targeted math concepts – symmetry**
(10:12) Peter (to the teacher): "how do you just have one line of symmetry? It's not possible."
Lisa: "Yes it is."
Peter: "Because all of the squares can be divided up in..."
The teacher walks over to Peter's desk.
Lisa: "Because if you divide it up here."
Peter: "Where?"
Lisa: "That would be the line of symmetry" (selects the select-a-grid horizontal line). "I can't divide it diagonally."
Lisa continues to explain, but her explanation isn't immediately clear to Peter.
Peter: "do you have to make it the same on each side? Is that what they mean?"
Lisa: "It can't have two lines of symmetry."

[GW4-4-15-04-t1][Distraction][EDIFF]
[PER]
-**Focusing on working on a design, even in the face of distraction**
(11:55) Peter asks Lisa again about the one line of symmetry challenge. Lisa explains, but has him wait until she is finished with her design. Details follow:
Peter (to Lisa): "So wait, I don't get it. How can there be only one line of symmetry?"
Lisa: "hold on" as she finishes working on her design.

[GW4-4-15-04-t1][DQT][grid][Symmetry][of a design]
[DD][Symmetry][of a design]
[WLB][Symmetry]
Peter: continues to watch Lisa and says, "Oh, it's only half?"
Lisa: "You can only fold it one way. All the other ways don't match up."
Lisa completes her design as Peter watches. Her design has exactly one line of symmetry.
Lisa displays the horizontal grid line and then the vertical grid line and tells Peter that the vertical would not work because the two halves wouldn't be equal. (See Figure 52).

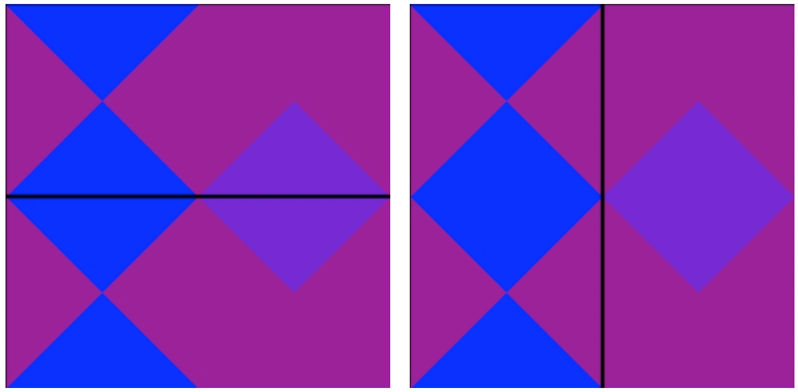


Figure 52. Lisa displays her quilt using two different gridlines to show a line of symmetry and line that does not work as a line of symmetry.

Peter: **"oh, now I get it!"**
Lisa: "and it can be diagonal", looking at Peter's screen. This comment seems to have come from looking at Peter's design so far. Peter clears his design without saving. He begins a new design.

[GW4-4-15-04-t1][DD][Design][How a design connects to something in the world]
[Naming][Design][How a design connects to something in the world]
[Sharing][STACCA][Design]

Naming quilts something from the world/finding real world things that look like a design

(17:15) Peter says his design looks like a frog (see Figure 53). Lisa agrees and says that she sees it too. This conversation continues. They compare their designs and discuss how they are similar.

Peter: "This looks kind of like a frog. I don't know why."
Lisa: (looking at his design) "It does. I can see it."
They have a conversation about how Peter's design looks like a frog.
Peter: "You can have, like, a thousand-frog army as a quilt."
Lisa chuckles. She does not mention the fact that his design is similar to hers.

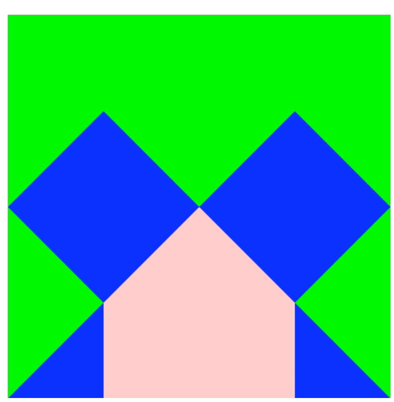


Figure 53. Peter's frog design – his final design with "exactly one line of symmetry."

Peter was so excited about solving this challenge. He seemed to need to let people know how confused he had been as a way to explain how few designs he had done so far that day. For instance, he explained to me in great detail how long he had struggled and how he eventually had to start over with his design. Looking at the video, he had only placed shapes in half of the design, but it would have been much more difficult to complete that design with a line of symmetry because the line would have needed to be diagonal if he was not going to have to delete anything.

<u>Different supports for symmetry bridging – examples and analysis</u>

The following example shows Ramona making a complicated quilt block design with many small squares in it. She uses the select-a-grid tool to find and describe the symmetry of her design. Ramona is working on a quilt block that is supposed to have at least three colors and at least one line of symmetry. It has lots of small squares. I ask her to tell me about her design. When she talks about the challenge she is working on, I ask her to show me how her design solves the challenge. She uses different grids to look for the symmetry in her design and correctly figures out which line would most easily work (none of them work yet since the design is not finished).

Ramona shows her symmetry plans

> [GW4-4-29-04-t2](0:00-1:44) remove this tag
> (0:20)Ramona: Explains which challenge she is working on – at least 3 colors and at least one line of symmetry. "I'm doing different colors of reds."
> (0:30) KK: "Can you tell me which line of symmetry you're trying to do?"
> Ramona: "Um, you can do right here, and right here (gestures to draw a vertical and then horizontal line with her finger). "It doesn't matter because [the challenge] says 'at least.'" Ramona is still adding shapes.

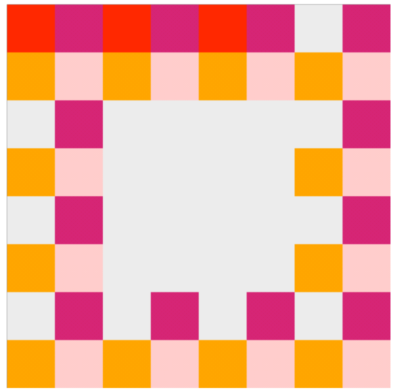


Figure 54. Ramona's design when she raises her hand to show me what it looks like.

(0:42) KK: "And, where do things match up? So, let's go through this one. If you fold it across this line of symmetry, where does that orange square land?" I am asking her where the squares of the design match up if folding over the horizontal line of symmetry. (I gesture to show the line I'm referring to rather than stating it verbally).

(0:49) Ramona says, "Let's see," and selects the horizontal line as her grid (see Figure 55). She talks about how the squares don't match up over the horizontal line. "That lands up here in the red box, so that won't work."

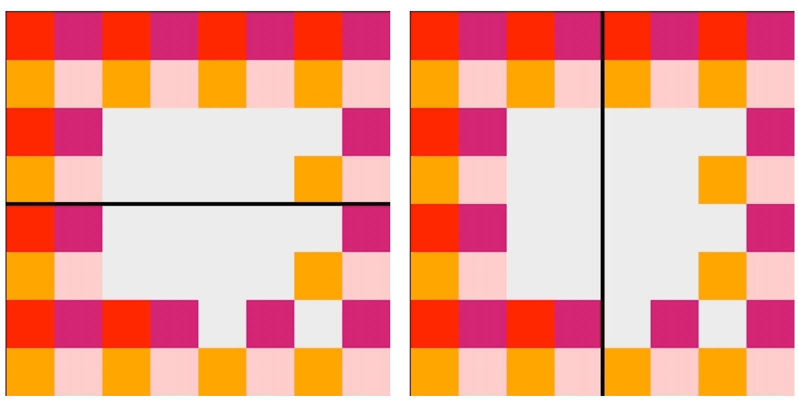


Figure 55. Ramona shows me her design with two different lines of symmetry selected.

(1:00) She changes the grid type to the vertical line and points to a purple square in the corner, "and that would land on that one," (a red square).

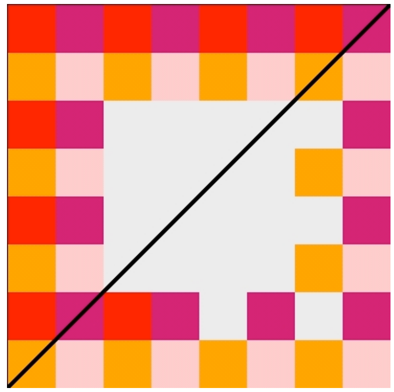


Figure 56. The line of symmetry Ramona thinks works. This is the line that is the closest to working.

> (1:09) Then, she selects a diagonal line. "And you could do it like… that I think… wait. Yeah, you could. [She looks across the diagonal line and seems to concentrate on the orange squares.] And…" She selects the other diagonal line, and then returns to the grid with no lines dividing up the block work area. In reality, none of the lines works yet, but if she continues placing red squares as she has started placing them, the diagonal line that she originally selected could be made to work (it is the only one that has the possibility of working without changing most of the design).

In this example, Ramona used the grid lines both as a way to check for symmetry in her own design and as a way to share her thoughts with me. In this case, the tools in the software seemed to play a role in helping Ramona make the connection between the idea of symmetry and the symmetry of her design. The way she completed her design, there were not any lines of symmetry (see Figure 57).

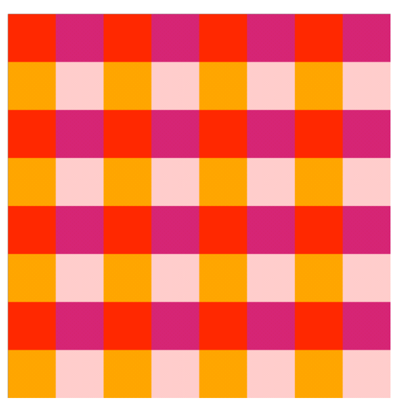


Figure 57. Ramona's finished design named "Stripes."

Though I imagined that the select-a-grid tool would provide some useful options for students to design symmetric quilt blocks since they could use the grids as points of reference, I did not necessarily realize how far some students would take use of the grids. In the previously mentioned example, "Peter's symmetry confusion," Lisa first tried to gesture to support her verbal explanation of how her design only had one line of symmetry. When this did not work to explain the challenge to Peter, Lisa used the grids in the select-a-grid tool to show a sample line of symmetry and to show that another line was not a line of symmetry. I had not anticipated that the students would use the grids to support their explanations, so this was a pleasant surprise. Ramona used the grids in a similar way, but in her case it was not in order to help me learn about symmetry but to help her explain her symmetry reasoning.

Joanna checks for symmetry

> [GW4-4-22-04-t2](25:53)
> The following example about checking the symmetry of a design, Joanna overhears me talking with Edzier about looking for symmetry in his design. Edzier's design is rather complicated and while his larger pieces match up across a line of symmetry, the details are not always quite correct. Joanna finishes her design and then appears to check for symmetry in her design by holding her hand over half of the quilt block on the screen and then doing a folding motion to see where parts of the design will land. She nods her head and seems satisfied that her design is symmetric. She moves on to another design.

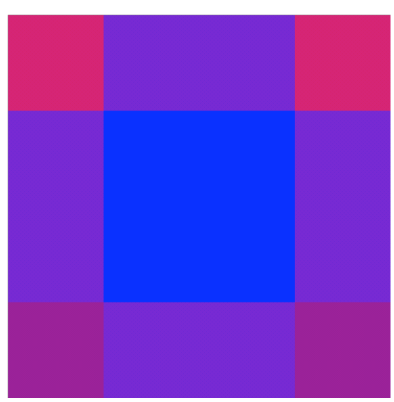


Figure 58. Joanna's symmetric design – "Fluffy #2" – named after her pet.

In this example, I cannot be sure exactly what enabled Joanna to connect her design to the abstract notion of symmetry. It appears to be the case that Joanna was trying to make a design that had a line of symmetry, that she checked for symmetry before moving on to another challenge, and that she successfully found the line of symmetry in her design. She wrote the name of this design on her challenge sheet in the spot for the design that had 4 colors and at least one line of symmetry. The social aspect of the socio-technical system definitely came into play here for Joanna – she was able to indirectly benefit from the guidance of a teacher-figure. However, Joanna's later attempts to verify symmetry are less successful (these will be described soon).

In the following example, Imani seeks verification from his classroom teacher that the design he made has exactly one line of symmetry. Imani has asked several other people about constructing this design along the way, and now that he is done, he shares it with his teacher. He asserts that the design has one line of symmetry, the teacher asks him to show how he knows, and he successfully shows her.

Imani seeks verification

> [GW4-4-15-04-t2](5:52)
> Imani: "Ain't this exactly one line of symmetry?"
> Ms. S: "Why is it one line of symmetry?"
> Imani: "Because you only can fold down. You can't fold over this way, 'cause if you fold over that way, you'll have two white spaces right there, so it's exactly one line of symmetry."
> Ms. S: "You're right."

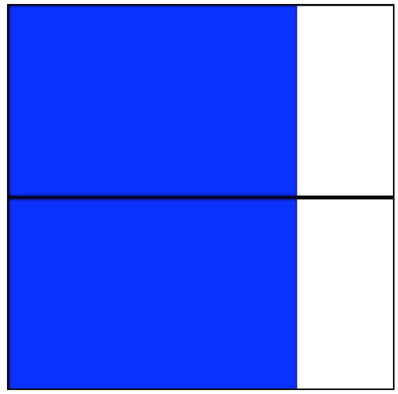


Figure 59. Imani shows this design to his teacher to verify the line of symmetry. He names it "1 line" because it has one line of symmetry.

In this episode, Imani had been struggling with symmetry. He seemed to take this opportunity to not only test his knowledge, but also to share his accomplishment with the teacher. He seemed to take pride in being able to show her how he knows that the design has only one line of symmetry. This kind of verification was more common for some students than others. In one episode on a different day, Edzier simply asks if his design solves the challenge, I ask him to show me the line of symmetry, and when he sees that there is not one, he clears his work (saving the design without giving it a name different from the default) and starts over again (see Figure 60). In this case, Edzier might not have made an example of a symmetric design, but he was able to determine that his design was *not* an example once he was prompted to check for symmetry – one example of social aspects of the socio-technical system at work. In each of these cases, there seemed to be a relationship between the specific design and the discussion or description of symmetry presented by the learner.

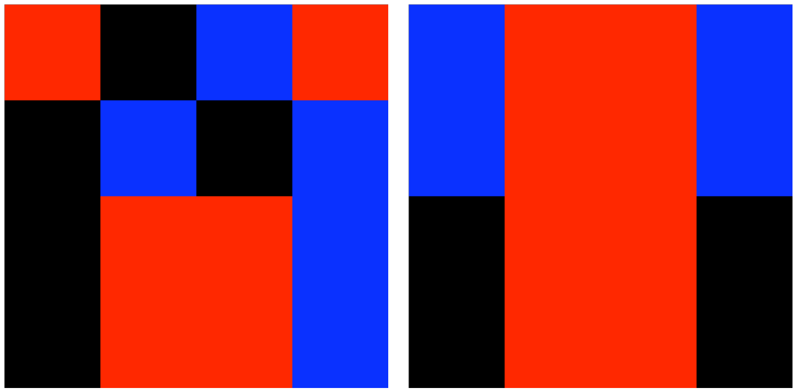


Figure 60. Edzier's two designs – one that did not solve the challenge and one that did.

In the following two related example episodes, Emma and Austin talk about symmetry. In one case, Austin asserts that his design *definitely* only has one line of symmetry. Emma, who is sitting next to him, asks where it is. Austin shows her how to fold the design. In the second example, about one minute later, Emma tells Austin that her design also only has one line of symmetry. He asks where and she shows him. Emma goes on to get Lisa's attention to tell her the story about how she didn't realize that her design only had one line of symmetry when she saved it, but now she sees that it does.

Emma and Austin discuss symmetry

> (25:32) Austin: "This definitely has only one line of symmetry."
> Emma looks at his design. (See Figure 61).
> Emma: "Where?"
> Austin: "You fold it in half like that. [He gestures with a folding motion.] See, you can't fold it the other way." [Emma looks on and then looks back at her own computer.]

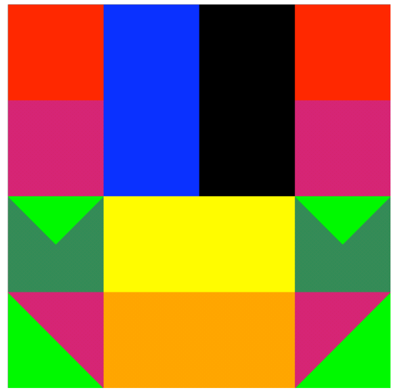


Figure 61. Austin's design he thinks has exactly one line of symmetry – "Thing 4"

> (26:24) Emma says to Austin: "Hey, this only has one line of symmetry."
> Austin: "where?"
> Emma: "right there" and she shows him (see Figure 62).

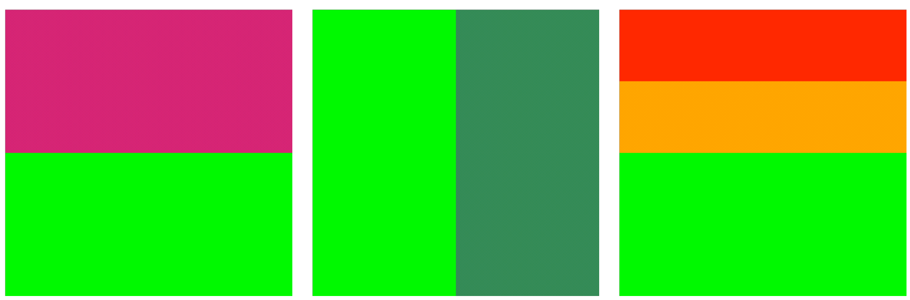


Figure 62. Emma's three designs with one line of symmetry – we can't see her screen to know which one she was referring to, but the one on the left is the one she used for the challenge she discussed with Austin.

> (26:34) Emma tells Lisa that she saved this one design that she didn't think had a line of symmetry. Then she went back and opened it again and realized that it does have a line of symmetry.

In these two related examples, the child making the design chose to get the attention of a neighbor to share about the symmetry of the design. In each case, the creator of the design asserted that the design had a particular attribute (in this case, exactly one line of symmetry). The creator saw the other child as part of some kind of audience (the creator of the design seemed to think that the audience member will care about the attribute for some reason). The creator of the design showed not only the design, but also this mathematically interesting aspect of it. In these examples, the student who was part of the audience asked the creator to share the claimed line of symmetry. What, exactly, helped the children show each other about the symmetry is not entirely clear in these examples, but the fact that the children were discussing the symmetry suggests that the presence and interest of peers played a role in the learner's attempts at finding symmetry in designs and creating symmetry.

<u>Software support for symmetry bridging</u>

Sometimes, the support for bridging came at least partly from the software tools I provided. Other times, the supports were less clearly from software tools, but the presence of the design within the software on the screen or on various printouts helped the children discuss their designs. The select-a-grid tool was utilized 9 times to support seeing or discussing symmetry.

In addition to two examples already listed (Peter's Symmetry Confusion and Ramona Shows Her Symmetry Plans), there were several examples where the grid was used to help children show the lines of symmetry they were talking about. In one example, Imani shows Douglas his design and tells him it has one line of symmetry. Imani explains the symmetry of the design in terms of folding. Douglas suggests using the grids to help them check.

Douglas suggests the grids

> [GW4-4-15-04-t2][(5:16)]
> Imani: "One line of symmetry. 'cause you can't fold it that way, you can only fold it this way." He gestures to the screen of his (now turned towards Douglas) computer. His gestures indicate the correct ways of folding. (See Figure 63 for the design.)
> Douglas [Douglas points at another grid]: "Click on it that way. Click on all of them [the grids]." Imani does this.
> Douglas: "Yay." [Both boys return to their work.]

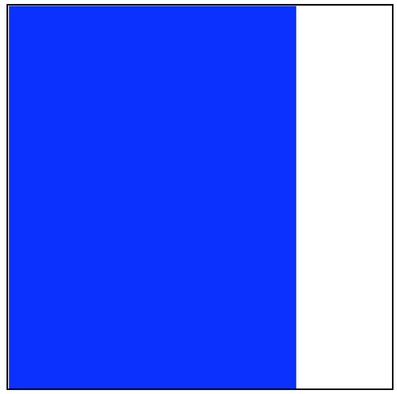


Figure 63. The design that Imani shows Douglas and tries all the grids to check for symmetry.

In this example, the student "audience" (Douglas) suggested using the select-a-grid tool in the software for supporting the claim made in the quilt block description. Imani obliged Douglas by using the grids. Douglas showed that he was satisfied that there was, indeed, a line of symmetry. In this case it seemed that the software supported the bridge between the abstract concept of symmetry and the concrete example (Imani's quilt block), at least for Douglas.

The select-a-grid tool was designed with fractions learning in mind, but students in GW4 used it more frequently for symmetry. The select-a-grid was the only tool in DigiQuilt that was designed with a symmetry-learning goal in mind (even a little). As such, more support for symmetry bridging seemed to come from other parts of the system.

<u>The folding metaphor as a support for symmetry bridging</u>

I used a folding metaphor to describe symmetry, showing the folding motion with my hands and talking about where each finger lands and how something like a ring may or may not land on another ring. The idea of folding and describing where things land came up more times than use of any particular software support. This metaphor supported the children as they discussed their designs in terms of symmetry. They asked each other to verify where the line of symmetry in a design was located, and they talked about where things would “land” if they were “folded.” The designs were not actually folded since they were on the screen, but the gesture of folding seemed to act as a guide for imagining what would happen if the designs were actually folded.

The metaphor of folding and the gestures associated with describing a design in terms of folding became an important part of the socio-technical system. Just as the quilt blocks themselves were useful as points of reference, the folding metaphor gave students another way of thinking about symmetry (besides just a mirror reflection). The gestures became part of a shared language and mode of interacting with the designs – a useful frame of reference. However, the system was not failsafe. Students still had some difficulties detecting symmetry in their designs.

<u>Experiencing difficulty connecting the abstract idea of symmetry to a concrete example of symmetry – connection not made</u>

The examples I have chosen for showing children making the connection between the abstract idea of symmetry and their quilt block designs include episodes that were especially *representative* or especially *compelling*. However, there were also examples where the children seemed to experience difficulty understanding the concept of symmetry, or at least connecting it to their designs. There were 7 episodes that I specifically tagged as times that children seemed to experience unresolved difficulty with symmetry. This does not include times that children successfully asked for and received help about symmetry. The cases with students experiencing difficulty generally were times when I would have liked to have seen the student ask for help, but perhaps the problem was not seen or understood by the child.

In this, the first example of an undetected misunderstanding of symmetry, Joanna removes all of the pieces from her second attempt at her third design (which she hasn't saved yet). The design she deleted had several lines of symmetry and was quite aesthetically pleasing. The progression of her designs can be seen in the section about what happened when learners experienced difficulty with symmetry. Once she starts over, she begins again by adding triangles of the same size and orientation and color to each patch. Then she adds a triangle of different orientation and color to each patch, creating a square with violet and pink on each patch. She checks for symmetry and, once she is satisfied it is symmetric, she moves on.

Joanna's misunderstood symmetry

> [GW4-4-22-04-t2][(35:18)]
> After working for about 3 minutes on the design, Joanna holds her hand up horizontally and vertically, checking for a line of symmetry. You can tell by her gestures that she is trying to picture folding the design over and seeing if it matches up. Her design does have a diagonal line of symmetry. However, it seems that she thinks it has a vertical line of symmetry, as she nods her head, saves her design, and clears it before moving on to the next challenge.

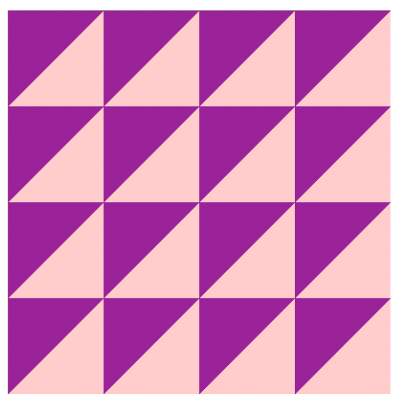


Figure 64. Joanna's design – "#3 Fluffy" that shows 8/16 and 8/16 and has one line of symmetry, but not the line she seems to think it does.

In the previous example, the video record was the only one that would show us that Joanna did not understand the symmetry of her design. The quilt block did have a line of symmetry, so if we checked up on her by looking at her work, we would not realize that she struggled. In fact, it was only by watching her work through the problem and check for symmetry that I know there was any confusion at all.

In fact, that was not the first time that Joanna experienced some difficulty with symmetry. In this example from just minutes before, she is working on a design that she clears because she does not see the line of symmetry.

Joanna misses the symmetry

> [GW4-4-22-04-t2][(28:45) (30:15)]
> (28:45) Joanna begins work on her fifth design, building on the 16-patch base-block. She adds triangles in different orientations. The triangles form two diagonal lines of symmetry. The way she is constructing it, she seems to add 1-3 triangles and then their reflections. "Wait a minute, is that one line of symmetry?"

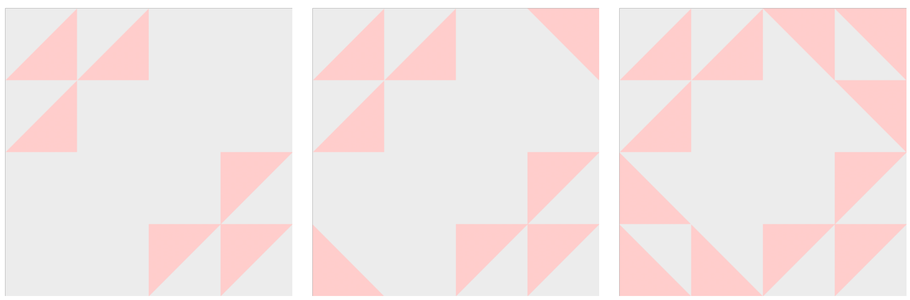


Figure 65. The progression of Joanna's design – she deleted this without saving because she did not see the line of symmetry.

> (30:15) Joanna checks her design for a vertical line of symmetry, holding her hands along the vertical axis and making a folding motion. She says, "Ok, that's not going to work." However, there is a diagonal line of symmetry that she doesn't pick up on. She clears the design without saving. She begins again.

Even though Joanna seemed to be constructing her design very deliberately in a manner that would maintain symmetry (adding shapes on one side of a line of symmetry and then adding the reflection to the other side, switching between the two sides frequently), she did not seem to understand that she was doing so. In the third example of experiencing difficulty with symmetry, Emma is using the horizontal grid line to construct her design. Her actions suggest that she is trying to use the horizontal grid line as a guide, but that she does not attend to local violations of symmetry in her design.

Emma does not notice symmetry violations

Emma selects the horizontal line as her grid before beginning her design. Emma fills in the top half of her block-work-area first. She places two red squares in the bottom half, removes one and clicks undo to get the other one to go away (she is trying to make a design with a line of symmetry, and she seems to have realized that the two red squares needed to be placed differently for this to work out correctly). However, the small squares in her design are not going to match up quite right, and she does not seem to notice that. She finishes her design, changing the colors as shown in Figure 66.

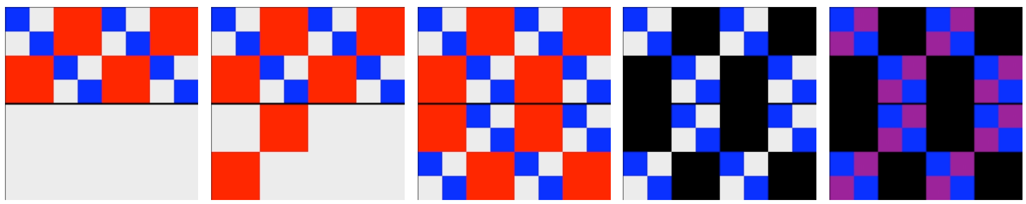


Figure 66. The sequence of images that show the steps Emma took to solve a symmetry challenge.

In the previous example, Emma used the grid to guide her, but the grid did not seem to be enough of a checking point to alert her to the fact that the smaller shapes did not line up across from each other like a reflection. She attended to the global symmetry in her design and ignored the local violations of symmetry.

All of these examples from the data showed students making connections between targeted concepts and concrete examples. I do not have data that shows that *each* student was able to make a connection (some students were not videotaped throughout an entire day of DigiQuilt use – only in short segments where they volunteered to show me something). However, the evidence I have presented comes from students with a wide range of interests and mathematical performance on tests. This evidence shows that a wide variety of students were able to make connections between the abstract targeted concepts of fractions and symmetry and concrete instantiations of these concepts while using DigiQuilt. In the next chapter, I will detail some possible explanations for the results of this study.

CHAPTER 8

DISCUSSION OF RESULTS

In the previous chapter, I described the results of my dissertation study including patterns that emerged in the video data and detailed descriptions of students' engagement with the design and mathematical aspects of their quilt blocks. In this chapter, I will describe possible reasons for the trends and patterns that emerged from the data (the disparity between design and math talk, and the quantitative difference between fractions and symmetry engagement). Then, I will compare the qualitative differences between the support that children seemed to need for their fractions versus symmetry talk and bridging.

Why were more interactions related to design than to bridging?

In my study, I found that there were many more times that children seemed to engage with the design aspects of the DigiQuilt socio-technical system than the targeted math content of fractions and symmetry combined. Because I am interested in supporting children's mathematizing *through* a design approach, I think it is important to try to understand what factors might have led to this disparity. I do not mean to imply that this difference is either unexpected or problematic, but I believe that understanding more about it could inform future research. Though the disparity between the number of times children's talk and activities centered on their designs versus the number of times they seemed to be engaging with fractions and symmetry might be described in many ways, here are some possible explanations:

1. The quantitative data depended somewhat on who was being videotaped on any given day.
2. Other strategies might have yielded different results.
3. The mathematical aspects of the quilt blocks were not always at the forefront.

4. Engaging with the design aspects of their designs was something the children came by without intervention, but engaging with the math required support – perhaps even more (or different) support than was offered.

Numbers depend somewhat on who is being videotaped

One possible explanation for the quantitative difference between design and math engagement in this study is that, on any given day, which participants were being videotaped could sway the numbers. That could mean that the numbers were impacted by my selection of focus students. For example, on the second day, the children discussed their designs (aside from discussing them in terms of fractions or symmetry) the least of any of the days. There were two cameras recording the video data from 4 focus students that day. Perhaps the most interesting thing to note about the children's talk about their designs on this day was that 15 of the times designs were discussed happened within one of the video taped groups, and only 2 happened in the other group. This suggests that the particular participants being taped on any given day can have a big impact on the quantitative data. If each day I had videotaped two groups like the group where design was not discussed often, the numbers could have been quite different – perhaps with children engaging with the math more often. This explanation seems unlikely, though, because I chose students with a wide range of interests and talents, and even if the numbers would have been different, the trend was very strong and did not seem to depend much on who was being taped.

Other strategies may have supported more math talk

The example from the previous explanation also brought up another possible explanation – namely, that there were most likely ways to encourage "math talk" and exploration that were not used in this study. Perhaps there were other ways to encourage math talk that were not explored in this study, but that would have had an impact on the quantitative data and the children's overall experience. Other strategies for helping the children engage within the socio-technical system might have led to more sharing.

Maybe sharing leads to more sharing. Or, maybe a certain kind of supported sharing leads to more sharing along the same lines. That might mean that it would be helpful to find a way to get each participant into a "sharing mood" at the start of each day – perhaps some kind of warm up activity. If the goal is to get the children looking at and thinking about their quilts in a mathematical way, maybe it would be useful to have them start each day by looking at a previous design and telling someone about the math of the design.

If the students could model for each other the process of pulling the mathematical nuances from their designs, that would probably be beneficial to their learning (e.g., if the students engaged in the sort of cognitive-apprenticeship described by Collins et al., 1989). Previous research suggests that having teachers and peers model content-related talk (Ryan & Kolodner, 2004) helps develop their understanding of those concepts and may even help them transfer their conceptual understanding to new situations. Having teachers externalize relevant processes involved in various cognitive skills (Collins et al., 1989), and then having students engage in successive approximations of mature practice helps the learners engage with content more like an expert – beyond working to gain domain knowledge, students can learn other expert strategies to help them learn more, carry out tasks, or try different approaches. Even providing a selection of prompts can help get them started on offering meaningful feedback to their peers (e.g., the prompts in CSILE), and "growing-into" the ability to engage in meaningful exchange of ideas (Scardamalia & Bereiter, 1991). All of these strategies or some combination could lead children to the point where real content gets explored, discussed, and hopefully understood more deeply. Perhaps if there were specific support in the socio-technical system for telling mathematical "stories" about the quilts, the learners would be better able to discuss the math in their designs. This explanation seems more likely than the first one because although there were some supports that could help the children as they talked about the math in their quilts, there was no specific, tangible support for getting them started in their collaborations, and only some attempts at modeling strategies for discussing the quilt blocks in terms of fractions and symmetry.

Mathematical aspects of the quilt blocks were not always at the forefront

Another possibility for the disparity between design and math engagement is that the mathematical aspects of the quilt blocks were not at the forefront as often as the designs themselves (which are almost always the center of discussion or engagement for most children throughout their design experiences). On day 6, there were 2 episodes related to fractions, 3 related to symmetry, and 22 related to design. Opening designs made by other students and making small changes was a popular activity that day. Since choosing designs made by other people involved a fair amount of browsing, it seems likely that the children spent more time admiring designs and less time looking at the math of the designs. If I had asked them to choose a design that had a certain mathematical criterion, that might have encouraged them to take a more mathematical look at the designs. The block browser shows the quilt blocks that students made, but not the fractions feedback or any grids that were used. The block browser includes an image of the quilt block, the name of the student who made it, and the name the student gave the quilt block. The mathematical aspects of the quilt block are still present, but they are not highlighted by the software when students are browsing or simply admiring printed designs.

This explanation seems related to criticisms of Froebel's gifts – the gifts embodied many mathematical properties, but a child playing with the gifts without any other intervention would not necessarily learn very much. The role of the teacher was emphasized. In DigiQuilt, the challenges were what brought these targeted concepts of fractions and symmetry to the forefront. The tools in the software were meant to further highlight the mathematical properties of the designs. More could have been done to highlight the mathematical features of the quilt blocks, especially those features that related to the targeted content. This explanation seems to account for times when there was not as much math talk when comparing between days – when design was the main activity and not searching for math in the designs, children were less likely to talk about math.

Engaging with design came more easily, engaging with math required more support

One other possible explanation for the gap between engagement with design and engagement with math is that the challenges seemed to play more of a *role* in promoting math talk, and less of a role in stimulating design talk and goal setting. As mentioned briefly in the previous paragraph, the challenges seemed to play an important role for helping the children focus on the math of their designs. No similar support seems to have been needed to help the children focus on the *design* aspects of their quilts – it seems to have happened rather naturally. Engaging with the design aspects of their designs was something the children came by without intervention, but engaging with the math required support – perhaps even more (or different) support than was offered. This points to a possible reason for the gap between design and math engagement – since the challenges were so important, maybe the challenges were not the right challenges to lead to math engagement or maybe there were not enough challenges.

An illustration of the importance of challenges for supporting engagement with the math happened on the last day of DigiQuilt use. Perhaps the most interesting thing about day 7 was that there were not *any* fractions or symmetry episodes. The students did not have any particular challenges to complete that guided them towards these mathematical ideas. However, the children did create design (that is design as in design/art/craft) goals for themselves, share those goals with their peers, and set out to accomplish their goals discussing their progress along the way. There were 36 episodes related to design, many of which involved children setting and announcing goals for themselves, or sharing the results of their efforts. This suggests that the challenges were not essential for children to engage with the design aspects of their quilt blocks. In other words, the learners were able to make personal connections even without the challenges. There seems to have been enough challenges – few children finished all the challenges on any given day. And, the challenges that were there did lead to mathematical discussions, so this explanation seems unlikely to account for the disparity on its own. It is possible that the challenges could have been better aligned with the software tools (so that we could have seen more about how the children were engaging with math in their designs) or supported in some other manner. The challenges seemed to be an important part of helping children leverage their epistemological connections.

Conclusions related to the differences in quantitative data between design and math

While this trend was striking, it is not the most important point that can be made using the data from this study. This last explanation suggests something more interesting than the other explanations combined. Perhaps the interesting story here is not that the children engaged more often with design than with math, but that they engaged with the math at all. The fact is that children were able to engage with both math and design in this socio-technical system, and that they could engage with the math without leaving behind the motivating nature of the design environment.

Why more interactions relating to symmetry than fractions?

Symmetry was the topic of student interactions approximately twice as many times as were fractions. Though the disparity between fractions and symmetry might be described in many ways, here are some possible explanations:

1. One student’s eagerness to discuss some particular thing might skew the results since I was recording a small sample of students on any given day.
2. The students were not challenged by the fractions in the challenges and felt no need to discuss them with anyone. (Or, symmetry was harder than fractions.)
3. The students could not understand the fractions well enough to talk about them. They were so confused that they did not know what to talk about.
4. The order of the challenges on any given day might have skewed the numbers if students did not get through all the challenges.
5. The challenges from previous days might have helped students prepare for certain challenges but not as well for others, prompting more discussion about challenges that were novel.
6. The children found symmetry more interesting than fractions.
7. The children thought symmetry was easier to talk about than fractions.
8. The feedback about fractions and symmetry offered by the software differed in a way that impacted both the quantity and type of “math talk” that occurred.

Who was videotaped and their discussion and activity preferences

One possible explanation for the disparity is which students were videotaped and what those students wanted to discuss could have a big impact on the numbers for any given day. This is part of the reason it was important to rotate to some degree who was recorded on any given day, and illustrates the impact that choices of target students can have on quantitative data.

For example, on day 2, 12 of the 19 symmetry episodes involved Imani, who seemed both eager to share his new knowledge about symmetry and also somewhat unsure that he understood it completely enough to move on to other designs without seeking verification from several people. I often used a folding metaphor for reflective symmetry with the students that involved gesturing with my hands and talking about where each finger would land. Imani adopted this metaphor quite readily and described his designs using gestures and talking about would happen if his designs were folded across a particular line of symmetry.

On day 4, there were 6 fractions episodes, 9 symmetry episodes, and 34 design episodes. Though I captured three children's experiences trying to understand 15/32, I was recording Joanna and Wendy with one camera, and found that they did not discuss fractions even once. The symmetry episodes were more spread out – involving 8 students that day. The design episodes were fairly evenly distributed that day. It is possible that recording different participants on that day could have yielded a different distribution. However, the fact that there was a fairly consistent trend favoring symmetry talk suggests that this factor was not the main reason for the quantitative disparity.

This is one explanation that would not apply as much at the other school. Because I had fewer students using DigiQuilt at any given moment at the other school, who was being videotaped at any given moment would not skew the numbers – each student was being videotaped almost all of the time they were using the software. Also, at CH Elementary, the students were encouraged to talk to the camera about their design activities or the math in the challenges they were working on (Lamberty & Kolodner, 2005). It was not logistically reasonable at GW Elementary to record each student all the time, so this practice of talking to the camera was not encouraged.

Maybe the fractions were not challenging enough?

The second possibility, that students were not challenged by the fractions, might apply to some of the simpler fractions. However, the level of discussion about some of the more difficult fractions suggests that at least some of the students were challenged by the fractions. The fact that these discussions often occurred between students who were generally high achievers in math (Emma and Lisa) also suggests that most of the students found at least some of the fractions difficult to understand without some effort.

Maybe the learners were overwhelmingly confused?

The third possibility, that the students were so confused they did not know what to talk about regarding the fractions, seems unlikely. Even the lowest math performers were able to solve many challenges that related to fractions. Sometimes, they asked for help, and other times they did not need to ask for help to successfully complete the challenges.

Order of challenges

Another possibility is that not many students got to the trickiest fractions challenge on any given day. In fact, of the students I was videotaping somewhat directly on day 2, only Lisa and Emma made it to the fourth challenge (make a quilt block that shows ½, ¼, and 2/8, a challenge that decidedly focuses on fractions). Lisa seemed to use the select-a-grid tool to understand something about the fractions of that problem. She started using the windowpane grid and part way through her design, she declared, "Oh, that's *easy!*" Following that remark, she completed a design that solved the challenge within a couple of minutes. She did not discuss the challenge with anyone else, so other students were not aware of her strategy or her solution. Emma seemed to figure out the challenge without discussing it with anyone, but the video only captured her face not her screen, so I don't know what design strategies she used to correctly solve the challenge. Overall, this explanation alone does not seem particularly likely to account for the trend because the order of the challenges varied from one day to another. Certainly, it had more of an impact when the children were still learning to function within the DigiQuilt socio-technical system on the first two days.

Preparation from challenges on previous days

Another possibility for the difference, that challenges on previous days had an impact on how challenges for any given day were perceived, may have had an impact. For instance, on day 2, only about 4 students had completed the challenge involving symmetry on the first day; so, for most of the students their second day using DigiQuilt was their first time encountering a symmetry challenge in DigiQuilt. This explanation does not seem as likely for future days, though, since the symmetry challenges varied less than the fractions challenges, and if lack of preparation explained the difference on day 2, one would expect that difference to be reversed on later days.

Maybe the learners found symmetry more interesting than fractions?

It is possible that the children found symmetry more interesting than fractions. I did not ask students about this during the study. Even if they did not find it more interesting, it is possible that it is easier to see the symmetry of a design and that reason alone could make it more interesting.

Maybe the children had an easier time talking about/connecting quilts to symmetry?

Along those same lines of reasoning, it is possible that children had an easier time talking about symmetry. For fractions, the learners may have had more models or algorithms they have experienced, but maybe all that variety made it difficult for them to understand how to think about or talk about the fractions in their quilts. Stated differently, the students might have really understood the model for folding and checking where pieces landed. Since that was the only model we discussed, they may have been better equipped to talk about symmetry – with a model in their minds that they could apply to each of the quilt blocks.

Type of support in the socio-technical system

A fourth possible explanation for the quantitative disparity between fractions and symmetry episodes is that the types of support available impacted how often children did something observably related to fractions or symmetry. The kinds of support for fractions and symmetry bridging students found within the socio-technical system varied, and this variance seems to have impacted the interactions of students both quantitatively and qualitatively. The support for symmetry (the grids in the software, the metaphors for discussion that were presented, the challenges, and the quilt blocks themselves as an object to think with) was different from the support for fractions (another way of using the grids in the software, the fractions feedback, and the challenges).

Even where similar supports were available (grids, challenges, and quilt blocks as things to think with), those supports seemed to play a different role. The grids provided a point of reference for symmetry discussions, or a way to divide the quilt into parts for fractions. The challenges prompted the children to look for symmetry or fractions, but they sometimes provided a different representation for fractions that could lead to expectation failure. The quilt blocks as things to think with provided something to talk about and change for symmetry discussions, while for fractions the fractions feedback changed as a result of some interactions so the child could interact with the quilt and see how another representation changed as a result. There was no such feedback for symmetry within the software – nothing that changed when there was a symmetry violation.

The biggest impact this would have on the number of times that either fractions or symmetry would be discussed seems to be that different kinds of discussion were needed. For the fractions, students could often tell for themselves if they were right or wrong based on the fractions feedback. Also, their knowledge of fractions (that it involves breaking something into equal sized pieces) might have been easier to align with the software tools. Therefore, children could check for themselves without needing to discuss fractions to see if they were right. There were different kinds of support in the software for symmetry than for fractions. Because the symmetry supports (especially the use of the folding metaphor which included gestures and physical movement) seemed to lend themselves more to discussion than to self-checking, the social part of the system played a larger role. As a result, there were more symmetry related interactions captured on video since it is inherently simpler to capture those activities that are external. I don't think it is a problem that the supports within the system differed for symmetry and fractions, but I think this explanation accounts for a significant impact on the quantitative differences between them in this study since the type of support seems to lead to more or less externally noticeable engagement. This would matter more if my focus were specifically on promoting *collaborative* learning. Then, it would be important to help children externalize their wonderings about fractions so they would be better able to discuss them. For this study, I was just hopeful that they would be able to make connections between the targeted content and their designs.

Qualitative differences between fractions bridging and symmetry bridging

As I mentioned early in this chapter, one of the noticeable trends in the data was that symmetry was discussed more often, and was more often the obvious topic of student interactions within the DigiQuilt socio-technical system. I already attempted to describe the quantitative differences between these types of bridging episodes. Here, I would like to look at the qualitative differences between the bridging supports for fractions and symmetry.

The qualitative differences seemed to be related to these four ideas:

1. What type of support was available?
2. How could learners use the support?

3. What kind of support was needed?
4. How do learners know if they need this support?

Different supports available

One difference between fractions and symmetry bridging was that different *kinds* of support were available. When children were discussing fractions, they were discussing the fractions in the challenge and trying to get that fraction. For fractions, the students had a larger set of possible algorithmic approaches to solving the problems (e.g., cross-multiplying to find larger fractions and multiplying the numerator and denominator by the same number to find equivalent fractions). They also had more options in the technical part of the system. The fractions feedback and the select-a-grid tool could be used in conjunction with each other and with students' algorithmic approaches to provide multiple ways to check the correctness of different solutions. For symmetry, this kind of checking was not possible even if the student would have sought it. When they were working on symmetry challenges, their feedback came from outside the system, so they needed to discuss their choices with people to find out if their approach was successful or not. Students needed to talk to each other to check for symmetry or seek verification for the correctness of their solutions. The social part of the system played a larger role. For example, when Peter wondered about symmetry, there was nothing in the software to tell him if he had one, two, or three lines of symmetry in his design. As a counter example, when Lisa was using the grids to find ¼, she was able to use the fractions feedback and the grids to figure out for herself that ¼ and 2/8 were the same.

How could learners use the support?

As has already been alluded to, even the same tool can be used in different ways for different things. One prime example of this was the select-a-grid tool. Another example is peer support: I can ask someone different kinds of questions or get help form my friends in different ways. In either case, the way learners could use the support may have differed for fractions and symmetry in a way that led to more engagement with symmetry.

<u>Using social support in different ways</u>

Sharing my design with my friend might support me in that it motivates me to continue. The students seemed to strongly prefer intricate designs that either looked like something from the real world or showed interesting and complex patterns. That means that the quilts related to symmetry challenges might have been more likely to be shared, as the compliments could act as a boost to the learners' motivation.

<u>The select-a-grid tool for fractions versus symmetry</u>

I was particularly struck by the fact that there were only two times in the GW4 data that children seemed to be using the select-a-grid tool to help them construct or talk about designs with a particular fraction. This can probably be explained in part because of the selection of challenges that I gave to this group of students. Most of the most interesting challenges involving fractions did not include fractions for which the grids would be most useful. It is also possible that some of the challenges that *did* involve fractions for which the grids would be useful fell on the first day when the sound on my recording equipment was not working, so I may have missed an opportunity to see children interact with the grids in the way I thought they would.

One other possibility is that using the grids for fractions required the learner to abandon their design goals too much in order to utilize the support that they offered. The learners seemed to strongly prefer designs where the pieces were mixed up as opposed to grouped tidily into separate sections. One student in GW5 even asked to have a couple of designs removed from the class set because he thought they were so boring. At one point, as he browsed through his old designs, Kyle asked me to get rid of his designs named

“Darkness 5” and “Darkness 3” because they were too boring for him (see Figure 67). This sounded like a testament to the fact that as children develop more sophisticated styles and figure out ways to do exciting things, they want to be able to present that new front to their audience. Sorting the different colors into distinct areas of the quilt block in order to use the select-a-grid tool resulted in this type of “boring” design. In other words, it is possible that they did not want to use the grids because they were not able to both use the grids and create a design they found interesting. Maybe for some students, the cost of using the select-a-grid tool for *fractions* was too great.

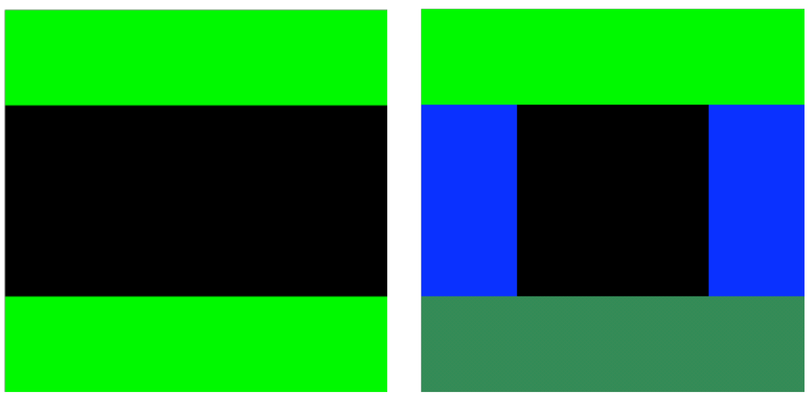


Figure 67. Designs that a student wanted deleted from the database of images because he felt they were too boring.

Whatever the reason for the lack of adoption of the select-a-grid tool for fractions learning, I see it as an interesting occurrence that merits further exploration. In future versions of DigiQuilt, I will change the select-a-grid tool to include a wider variety of fractions and align the challenges with the tool in a way that highlights its utility for fractions learning. Also, I imagine that there are ways to help the learners *create* their designs however they want to, and simply select a *view* that sorts the shapes into tidy groups that lend themselves well to using the grids (but in a way that allows you to go back to what you had). This would only be a temporary view on the design, but I think that it would enable the learners to make better use of the grids as a tool for understanding equivalent fractions without losing the motivation afforded by the design approach. It would lower the perceived cost of using the select-a-grid tool for fractions because it would not force the learner to choose between creating a complicated design with intricate patterns *or* using the select-a-grid tool to help with a fractions challenge.

Different types of support were needed

I think that the types of support that students *needed* contributed to these qualitative differences. Previous research on manipulatives suggests that it is important for manipulatives to embody the mathematical concepts they are intended to help students understand. In DigiQuilt, I attempted to create software that would highlight the ways in which DigiQuilt designs embodied the targeted concepts of fractions and symmetry.

The types of support that were needed differed. When children were discussing symmetry, they seemed to use more gestures and talk about the folding metaphor. Though the students used metaphor on their pretests and in interviews to describe fractions answers (mostly cutting and sharing pizza), they did not do much of that with DigiQuilt. The fact that the students did not discuss the metaphors related to fractions as readily while creating DigiQuilt designs might have been that they needed a different kind of support for seeing the fractions in their quilts.

The metaphors for cutting and sharing things like pizza and cake lend themselves well to manipulatives that show a series of equally sized pieces, some of which are shaded or missing from a total shape such as a pie. In that case, the metaphors are helpful for understanding what is seen. In DigiQuilt, the pieces are accounted for in the feedback even if they are not equally sized or located in adjacent spaces. The learner must rely more on the computer to keep that connection constant if the design involves mixed up pieces.

The symmetry challenges might have seemed more related to the design aspects of DigiQuilt. The fractions were less obviously embodied by the quilt block designs. Therefore, the learners needed more direction to focus on the fractions in their quilts. The fractions feedback was always available, but the challenges directed learners' attention to the feedback so they could utilize it. Sometimes, even when symmetry was not part of the challenge (or even addressed by the student in an obvious way), the design that resulted was still symmetric. While each of those quilts also had fractional areas that were covered by different colors, the symmetry is more noticeable at a glance.

Learners noticed their need for support in different ways

Finally, the students would notice different things about the symmetry and fractions of their designs as a result of the different supports that were needed and available. It was much more common for students to experience difficulty with symmetry and not know about it than it was with fractions. However, since the students could notice and fix problems on their own for fractions more easily than for symmetry, they wouldn't necessarily talk about fractions more. This difference between fractions and symmetry relates more to the times students were unsuccessful in solving a challenge. For the symmetry challenges, it would be easy for limitations of symmetry perception to let a learner think the challenge was solved by a given quilt block (relying too much on their "noticing at a glance"). For fractions, the combination of the feedback and challenges set learners up for opportunities for expectation failure more readily.

Missed symmetry violations and misunderstood symmetry

Several children either failed to notice local violations of symmetry, failed to reflect parts of their designs that landed on the right patch but in the wrong rotation, or just plain failed to detect the appropriate line of symmetry in their own designs. I think there must be a way to help children check for symmetry that would make this problem less likely to occur. However, one of the major challenges here would be designing a way for learners to detect a lack of symmetry or know that they should check for some error.

The easiest solution would be to design a tool that creates symmetry (the learner selects a line or lines of symmetry and whenever a part is added, the design is updated in a way that maintains the symmetry). However, I'm not convinced that is the best solution from a pedagogical perspective. I have several ideas for supporting the learners if they decide to check for symmetry (mostly involving mirroring cursors that afford checking similarity across one or more lines of symmetry). But, this type of solution does not handle drawing learners' attention to the fact that they misunderstand some aspect of symmetry. With fractions, when the feedback doesn't match the fraction in a challenge, the learners' attention is drawn to that fact if they are looking to the feedback to tell them something. Coming up with a similar representation of symmetry will be difficult, but I think it is important to come up with a way to alert learners to their misconceptions as well as helping them learn the content.

Conclusion

The students talked more about design than fractions and symmetry combined. That is not particularly surprising – the research suggests that children will find the design approach engaging and that they need some help connecting their designs to targeted content like fractions or symmetry. That the students talked more about symmetry was interesting. The kinds of support that children needed for bridging for those different targeted learning areas were both quantitatively and qualitatively different. As described earlier, the socio-technical system supported fractions and symmetry learning in different ways, and that seems to be neither good nor bad – it just is.

Most manipulatives don’t include specific supports for both design and bridging, so these differences have not been present in one socio-technical system before. Part of what makes DigiQuilt unique is that it is a manipulative that provides support for both design and linking the concrete and abstract – the fact that it does both at the same time without forcing the learner to use the manipulative in a completely different way sets us up to see these differences in the ways the two activities are supported. The children were able to leverage the affordances of the design environment and still make connections between their design activities and the targeted content areas of fractions and symmetry. In the closing chapter, I will present the DigiQuilt socio-technical system as a new kind of manipulative, highlighting the theoretical roots of manipulatives and situating DigiQuilt’s connections to and differences from those roots.

CHAPTER 9

A NEW KIND OF MANIPULATIVE

I started with the idea that quilting would be good for math. I decided on an approach that utilized manipulatives for design because there was lots of literature that supported both approaches (the use of manipulatives for math learning, and the design approach to learning). To help children get the most out of their experiences, I wanted both opportunities for design and math learning to stand out to students. This approach was unique in that it combined both the design approach and an approach that showed connections between concrete and symbolic fractions – the traditionally “mathy” stuff with the “artsy” stuff. Other manipulatives have done these things separately, but by taking advantage of affordances for design and affordances of using concrete materials (manipulatives) that are connected to abstract representations, the DigiQuilt socio-technical system combined these two genres.

To show that DigiQuilt is a unique style of manipulative, I will pull out some properties of manipulatives of different kinds and describe some language we can use to talk about manipulatives in a meaningful way. I’ll begin with a dictionary definition of manipulative, describe some historical views on manipulatives, and combine that with some categories laid out in the literature to show how this manipulative relates to previous manipulatives and traditions. DigiQuilt combines the affordances of a variety of types of manipulatives in a way that leverages computational power to support both bridging between concrete and abstract and creating personally meaningful designs. What makes it unique is that, through the use of “lenses” and constrained modes of interactions, it allows you to do both without drastically changing the use of the manipulatives.

Discussion: A New Kind of Manipulative?

What is a manipulative? How do we categorize or think about different kinds of manipulatives? What are the affordances of different kinds of manipulatives? In order to characterize this new kind of manipulative, I have compiled some criteria that seem to be important descriptors or defining characteristics of manipulatives in general. I will describe how DigiQuilt follows and differs from these traditions.

What is a Manipulative?

There seems to be a variety of ideas about what makes something a manipulative – does it just need to be concrete materials? How is a manipulative for design different from other tools for design? The American Heritage Dictionary of the English Language defines manipulative as follows:

> **ma·nip·u·la·tive**
> *n.* Any of various objects designed to be moved or arranged by hand as a means of developing motor skills or understanding abstractions, especially in mathematics.

Based on this definition, we can pull out some important characteristics of manipulatives and describe how they apply to screen-based manipulatives. First, there needs to be some idea of what it is that will be arranged, or the *objects* that comprise the manipulative. For a screen-based manipulative, the objects can only embody certain aspects of their physical counterparts (mostly, shape and appearance). Second, there needs to be *a way for the objects to be arranged*. The screen-based objects themselves usually look very similar to their physical counterparts, but the interaction with the objects will be different from interaction with the physical manipulative. For a screen-based manipulative, the objects can only be arranged in ways prescribed by the designer of the application. Third, the objects and their arrangement need to *help the learner understand one or more abstractions*. In my opinion, it is in this category that the designer of the screen-based manipulative has much more power than the designer of a physical manipulative.

There is more than one tradition in physical manipulatives for handling the way these three properties of manipulatives interact. Both Montessori and Froebel believed that children would learn best by doing things rather than focusing on observing the actions of others or understanding words. Though Montessori and Froebel both believed in the importance of the self-activity of the child, they had differing ideas about the role of the manipulatives and how they should be used. Manipulatives inspired by both of these traditions enable looking at relationships between objects and connecting the abstract and the concrete. It seems to me that not only the process, but also the result of using the manipulatives differs – with Montessori placing less emphasis on the resulting arrangement of the objects and Froebel emphasizing a design approach.

Theoretical roots – Montessori and Froebel

DigiQuilt is inspired by, but differs from, the manipulatives of both Montessori and Froebel. By utilizing the powers of the computer, DigiQuilt can be used to link these two different, but not incompatible, schools of thought about manipulatives.

In their paper about digital Montessori-inspired manipulatives, Zuckerman et al. (2005) describe two categories of manipulatives – those inspired by Montessori (Montessori-inspired Manipulatives or MiMs) and those inspired by Froebel (FiMs). They suggest that MiMs and FiMs differ in several ways. They say that FiMs allow children, "to design real-world things, objects, and physical structures." MiMs, they suggest, are also for building, "but focus primarily on modeling conceptual, more abstract structures." Their categories bring up some important points and echo criticisms of Froebel's gifts – which have been called both too simple to embody anything meaningful and too complex for children to understand (Liebschner, 1991).

I think their distinction misses an important difference between Froebel's and Montessori's approaches – namely, the roles of the manipulatives and the teacher. Both Montessori and Froebel emphasized the importance of a child's self-activity, but there seems to be a different role played by the teacher in supporting that activity.

From the Montessori perspective, DigiQuilt helps learners see relationships between the manipulative and abstract representations of targeted content (in this case, a numeric representation of fractions). However, it differs from Montessori and is more related to Froebel in that it allows the child to make these connections *while the results of their manipulations are combined to create a design*. This emphasis on the finished product does not seem to relate to Montessori at all. For Montessori, the emphasis would have been on the interaction process rather than the end result.

From the Froebel perspective, the use of a design approach in the DigiQuilt socio-technical system is not at all unique, but how the learners are meant to come to notice connections between representations is different. With Froebel's gifts, the burden of highlighting these connections seems to rest on the teacher. In DigiQuilt, the fractions feedback, the select-a-grid tool, and the other constraint-support structures (Kaput, 1992) or lenses help the learner see these connections. In Froebel's time, this computer-based support was not possible. DigiQuilt learners focus on creating interesting quilt blocks to explore mathematical concepts. From the constructionist perspective, this design focus seems important. The learner is not only working with a public, external artifact, but is working toward creating something that can be shared.

Interacting with Montessori's manipulatives

Montessori's manipulatives were meant to be self-correcting – the child would only successfully be able to use the manipulatives in one way (Montessori, 2004, p 58). The object's structure defined its use. A teacher's role is to provide a prepared environment with many opportunities for self-activity. The child has freedom to learn within that structure. A teacher's role, then, is to watch the child interact with the prepared environment, and, from there, figure out how to prepare the environment for the learner in the future based on what happens.

Interacting with Froebel's manipulatives

Froebel has been criticized for taking a similar stance – for saying the teacher should take a somewhat passive role in observing the child's interactions and letting those actions guide what should happen next. However, Froebel seemed to emphasize the role of the teacher in helping the child *notice* relationships between forms. His gifts were meant to be introduced in a prescribed way (thus attempting to suggest certain modes of interaction).

Though he was reluctant to prescribe every interaction that should take place with the gifts, he offered suggestions and guiding principles. One such principle was that new forms should be created from old forms rather than destroying old forms and starting from scratch. This would emphasize the connections between forms. He offered suggestions for helping the child progress through a series of interactions – first creating "forms of life" (creations that model things in the world), then "forms of beauty" (patterns with beautiful symmetries), and finally "forms of knowledge" (structures that emphasize mathematical concepts) with each gift. He believed that each child and teacher should have his own set of gifts so that they may each engage in meaningful self-activity. The importance of having a set of gifts for the teacher is that the teacher may demonstrate without interrupting the child's creations. The teacher's advice and influence can act as a lens for the learner.

Comparing and contrasting Montessori and Froebel

Both Montessori and Froebel emphasize the self-activity of the child (and this sets them apart from previous educational theorists), but there is a difference in how they envision the child making those connections between the concrete manipulative and the abstract mathematical concepts they are trying to learn. For Montessori, the manipulative embodies this math so completely that the child cannot fail to make the connection. For Froebel, the manipulative can embody many different things depending on the context of the use of the manipulative, and the teacher plays an important role in supporting the child during the self-activity.

The timing of the interactions between teacher and child are not so different – both believed that the child's self-activity and concentration were important and should not be interrupted. It seems to me (based mostly on Montessori, 2004, p. 270) that Montessori manipulatives themselves are more strictly structured to suggest a particular use and the child is meant to be free to use them how he pleases (though he will undoubtedly use them in a certain way), but Froebel manipulatives are not quite as auto-didactic. While the child should be permitted to use them in any way he chooses, a teacher might also demonstrate some possible interactions while asking some particular questions or making up some particular rhyme to help the child recall the interaction between the parts.

Froebel and Montessori both suggest free play, but both seem also to focus on asking questions or otherwise guiding learners to help them reflect on their experiences. Kilpatrick (as cited in Montessori, 2004, p. 274) says that Montessori is criticized for not having enough social interactions or collaborations for children, and Froebel for having too much collaboration coming from adults' considerations. In the DigiQuilt socio-technical system, collaboration is supported by the artifacts, the tools in the software, and the challenges. Free play is certainly possible with DigiQuilt, but challenges seemed to play a particularly important role in helping children get at the targeted math concepts (as evidenced in part by the lack of math talk on the last of day of DigiQuilt use by the students of GW4, and discussed in chapter 8).

While it might be useful to divide manipulatives into categories based on their inspirations, it is possible that this division could be made more useful by describing more properties of manipulatives that would fit into either category, and recognizing that there will not always be a completely clear distinction. For instance, the categorization of MiMs and FiMs fails to recognize that Froebel's gifts were meant to be used to create "forms of beauty" (patterns with beautiful symmetries) and "forms of knowledge" (structures that emphasize mathematical concepts) in addition to the "forms of life" (creations that model things in the world). The forms of knowledge seem less design oriented and more useful as a means of connecting concrete structures to abstract ideas than the other forms. Therefore, rather than saying that FiMs are for creating real-world things, objects, and physical structures and MiMs are for modeling conceptual, more abstract structures, I think that it would be appropriate to add that FiMs are designed with a particular role of the adult (or lenses and constraint-support structures) in mind, while MiMs try to embody the mathematic or symbolic content so that no particular lenses or modes of interaction are needed in order to reveal connections between the concrete manipulative and the abstract ideas it embodies.

Properties of Physical and Computational Manipulatives

Since new computational manipulatives are being developed, it is important to attempt to capture and communicate what lies at the heart of a physical or computational manipulative so that we can understand how to best design these learning tools. The properties of manipulatives that I have identified (objects, learning goals, modes of interaction, and lenses) manifest themselves differently in physical and computational manipulatives. These properties seem to capture similarities, as well as highlighting differences, among manipulatives. I hope that this list of properties will form a basis for the development of a series of design considerations for manipulatives – particularly computational manipulatives.

Most computational implementations of physical manipulatives have at least slightly different affordances than their physical counterparts. Even those virtual manipulatives that seem to *aim* to be a direct copy of their physical counterparts tend to change how the learner can interact with them including the kinds of arrangements that are possible and the kinds of actions that can be taken. Such differences can actually be helpful to learners. When objects snap in place in a grid, for instance, less dexterous users are likely to have an easier time making their arrangements of objects look tidy. In addition to changing how the learner interacts with the objects, computational manipulatives can be used to help focus the learner's attention in different ways. I made several design decisions in DigiQuilt that ultimately modified the learner's experience with quilting tiles in an attempt to help them learn more. Some of these changes affect the learner's experience by altering the way one can interact with the manipulative (modes of interaction – e.g., in DigiQuilt, the property that all pieces and patches snap into place rather than allowing learners to set them anywhere) and some by changing the focus of the learner's attention (I will call these *lenses* – e.g., in DigiQuilt, the select-a-grid tool that allows learners to superimpose their choice of grids on a design without rearranging pieces).

Objects

The objects that comprise the manipulative are the things that remain the most constant between a physical manipulative and its virtual counterpart. When it comes to manipulatives in general, some objects embody an abstraction more completely than others do. Base-ten blocks, for example, have objects that embody the relationship between ones, tens, and hundreds (see Figure 68), while simple counters could be used to hold place value but would not really embody anything about the place value in a meaningful way.

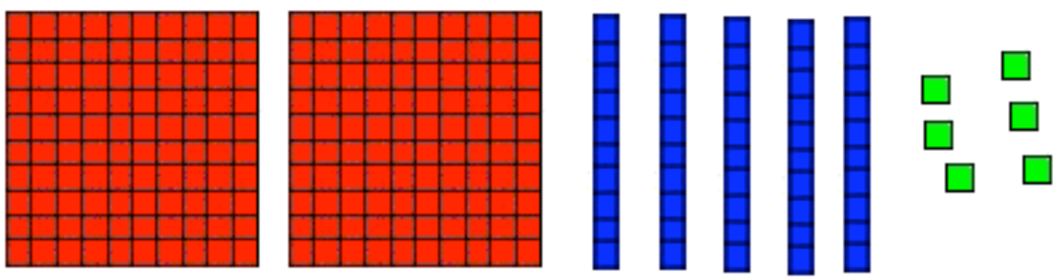


Figure 68. The number 256 represented using base-ten blocks.

The quality of the mapping between the object and the abstraction with which it connects can have a profound effect on how well the learner actually learns through using the manipulative. Random assignment of abstraction to concrete representation, then, is something to avoid because it can be difficult for learners to see the connections if the manipulative does not embody the concept completely enough. This is especially true of physical manipulatives or any manipulative where the modes of interaction are not constrained.

In DigiQuilt, I chose pieces that would fit together nicely to make quilt block patterns and provide nice proportions for talking about fractions. I made the same pieces available in the software that I made available as paper pieces in the physical version of the manipulative. While the objects chosen to embody the abstraction are important, they do not tend to vary much between physical and virtual manipulatives. Still, there are some aspects of the objects that comprise a manipulative that help the learner determine something about how the manipulative is meant to be used. For instance, if there are obvious ways to stack or connect different parts of the manipulative (as with unifix cubes), it is likely that the objects would be stacked or connected by the learner. If this sort of construction activity is useful for achieving a learning goal, then the object itself could be said to help the learner determine how to achieve that goal.

Learning goals of the manipulative

The objects of a manipulative play a role in determining how the learning goals of the manipulative will be achieved. What are the goals of the use of the manipulative? How can the objects be used to help the learner achieve those goals? Some manipulatives are more versatile than others and have a wide range of affordances. The similarities between things like Cuisenaire rods and fraction strips as far as form factor are fairly noticeable, but they have differing affordances. The fractions strips clearly focus on the abstraction of fractions and don't translate well to adding integers or thinking about negative numbers. They are less likely to be used for designing any kind of images or patterns than the Cuisenaire rods (though they may work for design, the labels on the strips may detract from people's perceptions of them as design elements). So, we can say that fraction strips have a more specific learning goal than Cuisenaire rods (or a narrower focus, depending on how you want to look at this). Further along that continuum, pattern tiles are more likely to be used to create interesting patterns than either Cuisenaire rods or fraction sticks. On the other hand, they can be used for fractions learning with some additional guidance. So, the *specificity* of the learning goals of using a manipulative varies. Some objects embody only one abstraction, while others may embody several mathematic abstractions and rely on other means for connecting them with those abstractions that are a farther match.

How the various learning goals are actually *supported* also varies. Support for achieving particular learning goals can vary in two ways: 1) what support is *needed* and 2) what support is *available*. How big is the gap between what the learner can see and what the learner needs to learn? The support that is needed is likely to vary depending on the size of the gap between the concrete and the abstract representation.

For example, pattern tiles are often used for exploring geometry and patterns. The gap between what the learner is trying to understand and the actual objects that are being manipulated is very small. However, pattern tiles are sometimes used to help learners understand fractions. In this case, the gap is larger, so the learner probably needs more support to see how the objects they are manipulating relate to the abstract representation of fractions. This bridge could be in the form of questions that draw the learner's attention to certain relationships between the pattern tiles (as can be seen in Lanius, 1997-2004), but it could also be supported through different modes of interaction or lenses in a computational manipulative. Activities that utilize pattern tiles for fractions learning that I have seen tend to have learners using the pattern tiles in a way that separates the fractions learning from the design activities that the tiles afford so nicely. That is to say, the activities don't enable learners who are thriving in a design approach to remain in their design setting and still learn about fractions.

Understanding more about how the objects embody an abstraction and how learners can learn by manipulating the objects can help us determine how to support the learner's interaction with the manipulative. The question for designers is: How big of a bridge needs to be built to cover the gap? Many design manipulatives are used for geometry learning (as described in the pattern tiles example). DigiQuilt helps bridge a slightly bigger gap: from physical object used for design to numeric representation of fraction. Therefore, the modes of interaction and lenses that are part of the sociotechnical system play a larger role than they would if the objects more clearly embodied the learning goals of the interaction. In fact, these lenses and modes of interaction enable fractions learning without forcing the learner to leave the design environment. This seems to be true for both Froebel's gifts and DigiQuilt – the design part of the system seems less likely to lead to learning of targeted content if there are no constraints or suggestions about how to use the manipulatives (mode of interaction), or if there are no challenges or hints to help learners notice certain things (lenses).

Modes of Interaction

Manipulatives are, "designed to be moved or arranged by hand as a means of understanding mathematic abstractions." Anything that limits or extends the interactions a learner has with the physical embodiment of a mathematical abstraction will have an effect on how the learner understands the abstraction. Restricting (or augmenting) modes of interaction can give learners a more accessible (or more advanced) learning experience.

Because the designer of the computational manipulative has more control over actions that can be taken by the user, modes of interaction are more easily controlled with virtual manipulatives than with physical manipulatives. In a computational manipulative, it is easy to imagine modes of interaction being used to highlight actions that can be taken on either the objects that comprise the manipulative or the abstraction embodied by the manipulative and seeing the effect of the action on the other representation (multiple linked representations). This is different from physical manipulatives in that the relationship between the concrete and abstract representations can be kept constant automatically so that the user can focus on the effects of the changes that were made rather than updating one representation to match the other (a time when mistakes are likely to lead to a poor understanding of the connection between the concrete and abstract representations).

In DigiQuilt, the learner's interaction with the objects is different than with physical quilting tiles. The shapes are presented in only one orientation and must be turned by the learner in order to use them in the design in any other orientation. In addition, the pieces and patches can only be turned in 90-degree increments. These restrictions make it so that the pieces will always fit into the provided grid. The pieces and patches also "snap" to the grid rather than forcing the user to align them exactly as they want them to be included in the design or allowing them to overlap in ways that are not defined by the system. These restrictions make it easier for the learners to find and share the math in their designs because the fractions resulting from this kind of arrangement are more predictable. Also, the limits to how the shapes can be placed in the design allow less dexterous users to make their designs look neat and tidy. To simplify the learner's experience, DigiQuilt can (optionally) be used with only squares and half-square-triangles (as opposed to additionally being able to use half-square-rectangles, or quarter-square-sized squares and triangles). In addition, there are 4-, 9-, and 16-patch base blocks for building different kinds of quilt blocks. In a physical version of the manipulative, learners can be instructed to use a particular grid or limit themselves to certain shapes, but in the virtual world, the other modes are available when the learner is ready and it is easy to switch between modes.

In addition to augmenting or restricting the ways a learner can interact with a manipulative, some objects and modes of interaction allow more than one kind of interaction. Besides helping connect quilting tiles to fractions, the quilting tiles in

DigiQuilt can be used for design. The activity of designing patchwork quilt blocks seems to be motivating enough for most students to keep them engaged with the manipulative. The design aspect of DigiQuilt also gives the learners a sense of audience. They feel that their designs are worth sharing and seem to have a sense of pride associated with completion of a unique design. Since the learners are always receiving some feedback from the system about what fractional area of their quilt block is covered with each color, even if they are not actively pursuing a fractions related goal they are able to peek at an abstract representation of their concrete design. DigiQuilt encourages exploratory learning because it is easy to make changes, see the results of those changes in both the concrete and abstract representations, and change back. Though this sort of exploration is not expressly prohibited in the physical version of DigiQuilt, the computational version makes these explorations much simpler.

Lenses

Sometimes the learner might not understand the connections that exist between the concrete and abstract representations because the connections are not easy to notice. In addition to augmenting or restricting how learners can *manipulate* the objects, I propose that lenses for viewing or examining the resulting arrangement in a new way can help them understand more about the relationship between the abstract and concrete representations by focusing their attention on particular aspects of the arrangement. These lenses can augment the learners' experience by helping them *interpret* their concrete designs in an abstract way without forcing them to completely abandon the design environment that many of them find so engaging.

"Lenses" is the one property of manipulatives that seems to be less universal. In conjunction with "modes of interaction," I believe that helping the learner notice certain aspects of what they are doing or the tools they are working with provides additional support. Lenses are probably the least common in physical manipulatives. I have seen some things like trays or varying sizes that are labeled with the number of unifix cubes they can hold, design cards with lines on them to guide students or provide puzzle-like settings for them to work within, and even workbooks. None of these is quite like "lenses" as I envision them for manipulatives. These supports seem to be geared towards changing the way children can use the manipulatives (i.e., modes of interaction) more than they help the children to interpret what they have already done or are doing. It seems to me that any tool that must be used throughout the design experience or before the learner begins engaging with the manipulative itself (choosing a tray for holding cubes or a design card) is not just providing a new view of the result. Maybe some tools can play more than one role, but I have been thinking of lenses as things that can be changed without changing the result of the interactions (in the case of DigiQuilt, the design itself). While lenses are not common in physical manipulatives, sometimes other factors come into play. For instance, in Froebel's gifts, the teacher's interactions with the learners throughout their engagement with the gifts acted as a lens of sorts, helping the learner notice certain mathematical aspects of their designs. Still, lenses (and multiple views on the same information) are more common in computational manipulatives.

Some computational manipulatives allow the user to interact with the manipulative through multiple views that highlight different aspects of the manipulative. For instance, HyperGami and JavaGami (Eisenberg & Eisenberg, 1997, 1998) allow the user to design their artifacts by either manipulating the image or by changing equations, but changing parts of the structure causes details in the design to be lost. The presence of multiple views is like a set of lenses on the same design. The fact that you can change the artifact through those views is unrelated to my notion of lenses (since it deals with actual interactions with the artifact, it falls more into the "modes of interaction" category). Part of the beauty of allowing the learners to interact with their artifacts without changing them is that it emphasizes the different ways of looking at the same thing – the quilt blocks themselves can act as a force of constancy. Highlighting the lack of change is helpful for learners' understanding, and it also allows them to explore multiple ways of understanding something without worrying about changing their designs. In JavaGami, there is no facility to undo. In HyperGami, users can choose to save out the solid and net version of their polyhedra before trying anything they foresee as "risky," but if the risk is not foreseen, things get tricky. In either system, if the user makes what he/she perceives to be a mistake, the only way to fix it is by using a mathematical way out (M. Eisenberg, personal communication). The mathematical sophistication required to accomplish this may be enough to cause learners to cease exploration. JavaGami and HyperGami give users some feedback about the math involved in their designs, but might not encourage learners to do a lot of exploration unless they feel comfortable that whatever they are doing can be undone.

Contributions

The work completed in the process of writing this dissertation contributes to the learning sciences and technology field in two ways. First, it offers suggestions for (and an example of) a new kind of computational manipulative, including a description of its use in the field. Second, it offers an example of a constructionist experience within a current classroom setting that was accessible to many learners, and allowed some learners to delve more deeply in their free time – perhaps discovering a new passion or hobby.

A New Kind of Manipulative

Some of the affordances of DigiQuilt are present in other manipulatives. I have described other manipulatives that are used for design, that are computer-based, that use the computer to provide constant feedback about some conceptual area, that utilize multiple linked representations, and that can be used to create lasting artifacts. DigiQuilt is unique in that it combines these properties in a new way. In particular, I have said that the kind of computational manipulative I have created:

1. supports learners by restricting the modes of interaction in response to the learning goals of the manipulative,
2. encourages or affords more than one kind of interaction (here, the learner might be pursuing a design goal, a fractions goal, or some combination of the two),
3. offers feedback to the learner about the connections between the concrete and abstract representations,
4. uses lenses to help the learner interpret the arrangement of objects
5. allows for easy exploration, and
6. simultaneously supports learners' efforts at making connections between the concrete and abstract and creating designs.

Combining the benefits of using a computer (keeping relationships between representations constant, giving learners feedback without intervention, and supporting mathematical interpretations of their arrangements of objects) with the benefits of a design approach (personal connections, motivation, and concrete examples) has all the signs of a successful learning environment. Using this software kept students (enthusiastically) engaged in a productive learning experience. Learners successfully engaged in mathematical discussions about their designs in the context of solving mathematical challenges. The learners cared about their designs enough to persist through difficulties (even when the difficulties were self-imposed challenges). This new manipulative has affordances for supporting both design and conceptual learning, and, most importantly, it affords realizing both sets of affordances at the same time. A learner does not need to leave the design environment to explore fractions, or leave the fractions support to engage in design.

A constructionist experience in a classroom setting

The second contribution to the field of learning sciences and technology is that I have described a constructionist experience that fits into a current classroom setting. The students who used DigiQuilt were doing so within the confines of their regular school day, though some of them chose to extend that experience into their free time. The elements of the socio-technical system described here allowed children to leverage personal connections (by making designs they cared about) and epistemological connections (by highlighting connections between their designs and the abstract mathematical ideas of symmetry and fractions through the use of challenges and lenses). In particular, the challenges played an important role in steering learners toward mathematical investigations. When there were no challenges, math was not discussed. The affordances of the system were not enough. Children needed cues from outside the technical system in order to leverage the math-learning affordances. The challenges steered the learners toward fractions and symmetry, but they could create anything they wanted to in order to satisfy the challenge. The delicate balance between freedom and structure was maintained – enough freedom for exploring personal connections, but enough guidance to steer learners toward epistemological demands of the curriculum.

Helping Children Mathematize their worlds: Future work

This new kind of manipulative seems to allow learners to leverage similar affordances to both design and learning with manipulatives. Eventually, I would like to extend the math learning goals and tools so that students could learn to look at their designs in many ways (through a variety of “math lenses”) and to mathematize a variety of situations. I’ve already created a mini-mathland for learners to explore math on the computer, and helped learners bring physical artifacts from mathland (quilt designs on business cards, magnets, and paper printouts) into their world.

Certainly, these are important steps toward helping children mathematize their worlds, but more remains to be done. This study did not look very closely at how much math learners retain about their mathematic designs once those designs are removed from the mini-mathland on the computer. I want to know more about how students may develop new “math lenses” they can use to view the world. I want to design more tools that allow learners to see the same artifact through a wider variety of lenses to be used in the context of challenges that ask them to *explore* these different perspectives – perhaps by integrating challenges into the software and connecting them more closely with different lenses. I want to figure out new ways to support learners’ mathematical memories of their artifacts and create interesting and lasting ways for them to share the math of their designs – perhaps by providing some kind of personal scrapbooking system with stickers and mathematical prompts that make it easy to jot down and remember the math of the quilts once they are away from the mini-mathland on the computer, or by creating math games involving the quilt designs that can be played away from the computer. I want to apply lessons learned from creating this design manipulative to create a new design manipulative. I want to help children learn more about how to mathematize everyday things so that math will be all around them in the world.

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